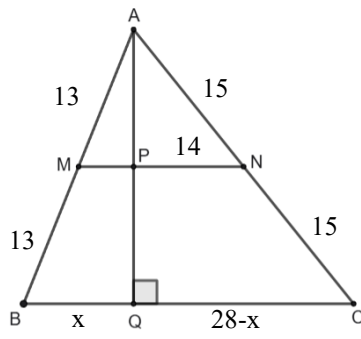
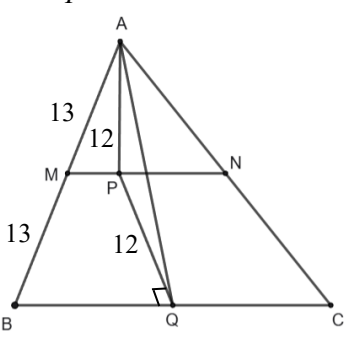


Olimpiada Națională de Matematică
Etapă locală, Galați - 7 februarie 2026
Clasa a VIII-a

Barem de notare și evaluare

- Pentru orice soluție corectă, chiar dacă este diferită de cea din barem, se acordă punctajul maxim corespunzător.
- Nu se acordă fracțiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parțiale, în limitele punctajului indicat în barem.

Nr. problemei	Soluție, rezolvare	Punctaj
1.	<p>a)</p> $x \in (-2, 6) \Rightarrow -2 < x < 6$ $y \in (-5, 3) \Rightarrow -5 < y < 3$ $\begin{array}{c} (+) \\ -7 < x + y < 9 \end{array} / -9$ $-16 < x + y - 9 < 0 \Rightarrow$ $\Rightarrow x + y - 9 = -x - y + 9$ $0 < x + y + 7 < 16 \Rightarrow x + y + 7 = x + y + 7$ <p>Deci, $n = x + y - 9 + x + y + 7$</p> $n = -x - y + 9 + x + y + 7$ $n = 16 = 4^2 \Rightarrow n \text{ este pătrat perfect}$	<p>1p 1p 1p 3p 1p 2p 1p</p>
	<p>b)</p> $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{2024 \cdot 2025} =$ $= \frac{2-1}{1 \cdot 2} + \frac{3-2}{2 \cdot 3} + \frac{4-3}{3 \cdot 4} + \dots + \frac{2025-2024}{2024 \cdot 2025}$ $= \frac{2}{1 \cdot 2} - \frac{1}{1 \cdot 2} + \frac{3}{2 \cdot 3} - \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} - \frac{3}{3 \cdot 4} + \dots + \frac{2025}{2024 \cdot 2025} - \frac{2024}{2024 \cdot 2025}$ $= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2024} - \frac{1}{2025}$ $= 1 - \frac{1}{2025} = \frac{2024}{2025}$ $\sqrt{x^2 + 2x \cdot \frac{2024}{2025} + \left(\frac{2024}{2025}\right)^2} = \frac{4049}{2025}$ $\sqrt{\left(x + \frac{2024}{2025}\right)^2} = \frac{4049}{2025}$ $x + \frac{2024}{2025} = \frac{4049}{2025} \Rightarrow x = \frac{2025}{2025} \Rightarrow x = 1 \in \mathbb{N}.$	<p>2p 2p 2p 1p 2p 2p</p>

2.	<p>a) I înainte de îndoire</p>  <p>Fie $AQ \perp BC$; notăm $BQ = x$ $\Delta ABQ \xrightarrow{T.P.} AQ^2 = AB^2 - x^2$ $\Delta ACQ \xrightarrow{T.P.} AQ^2 = AC^2 - (28 - x)^2$ $\Rightarrow x = 10$ $\Delta ABQ \xrightarrow{T.P.} AQ^2 = 26^2 - 10^2$ $AQ = 24$ $AQ \cap MN = \{P\}$ $AP \perp MN$ $AP = PQ = \frac{1}{2} \cdot AQ = 12$</p> <p>II după îndoire</p>  <p>Fie $AP \perp MN$; $AP \subset (AMN)$ $(AMN) \perp (MNCB)$ $(AMN) \cap (MNCB) = MN$ } \Rightarrow $AP \perp (MNCB)$ $PQ \perp BC$ $PQ, BC \subset (MNCB)$ } $\xrightarrow{T3\perp} AQ \perp BC$ $\Leftrightarrow d(A, BC) = AQ$ $\Delta APQ \xrightarrow{T.P.} AQ^2 = AP^2 + PQ^2$ $AQ = 12\sqrt{2}$</p>	<p>2p</p> <p>2p</p> <p>2p</p> <p>2p</p> <p>2p</p>
	<p>b) $A_{\Delta ABC(\text{înainte})} = \frac{AQ \cdot BC}{2} = \frac{24 \cdot 28}{2} = 336 \text{ cm}^2$ $A_{\Delta ABC(\text{după})} = \frac{AQ \cdot BC}{2} = \frac{12\sqrt{2} \cdot 28}{2} = 6\sqrt{2} \cdot 28 = 168\sqrt{2} \text{ cm}^2$ $\frac{A_{\Delta ABC(\text{înainte})}}{A_{\Delta ABC(\text{după})}} = \frac{336}{168\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$</p>	<p>3p</p> <p>4p</p> <p>4p</p>
3.	<p>a) $\begin{cases} \overline{abc} = \overline{xy^2} + \overline{uv^2} \\ \overline{cba} = \overline{yx^2} + \overline{vu^2}, a < c \end{cases}$ $\begin{cases} 100a + 10b + c = (10x + y)^2 + (10u + v)^2 \\ 100c + 10b + a = (10y + x)^2 + (10v + u)^2 \end{cases}$ $(+)$ $101a + 20b + 101c = 100x^2 + 20xy + y^2 + 100u^2 + 20uv + v^2 + 100y^2 + 20xy + x^2 + 100v^2 + 20uv + u^2$ $101(a + c) + 20b = 101(x^2 + y^2) + 40xy + 101(u^2 + v^2) + 40uv$ $101(a + c - x^2 - y^2 - u^2 - v^2) = 20(2xy + 2uv - b)$ $(101; 20) = 1 \Rightarrow \frac{101}{20} \mid (2xy + 2uv - b)$ $\frac{101}{20} \mid (a + c - x^2 - y^2 - u^2 - v^2)$</p>	<p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p>

	$\left. \begin{aligned} a + c &\leq 18 \\ x^2 &\geq 1; y^2 \geq 1; u^2 \geq 1; v^2 \geq 1 \end{aligned} \right\} \Rightarrow$ $a, b, c, x, y, u, v - \text{cifre nenule}$ $\Rightarrow a + c - x^2 - y^2 - u^2 - v^2 \in M_{20} < 20$ $\Rightarrow a + c - x^2 - y^2 - u^2 - v^2 \leq 0$ $\Rightarrow 2xy + 2uv - b \leq 0$ $2xy + 2uv - b \geq 2 \cdot 1 \cdot 1 + 2 \cdot 1 \cdot 1 - 9 \Rightarrow 2xy + 2uv - b \geq -5,$ $\text{Și cum } 101 / (2xy + 2uv - b) \Rightarrow 2xy + 2uv - b = 0 \Leftrightarrow$ $\Leftrightarrow \begin{cases} b = 2xy + 2uv \\ a + c = x^2 + y^2 + u^2 + v^2 \end{cases}$ $\xrightarrow{+}$ $\Rightarrow a + b + c = x^2 + 2xy + y^2 + u^2 + 2uv + v^2$ $\Rightarrow a + b + c = (x + y)^2 + (u + v)^2$	<p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p>
b)	$\begin{cases} \overline{abc} = \overline{xy^2} + \overline{uv^2} \\ \overline{cba} = \overline{yx^2} + \overline{vu^2}, a < c \end{cases}$ $\xrightarrow{-}$ $\Rightarrow \overline{cba} - \overline{abc} = \overline{yx^2} - \overline{xy^2} + \overline{vu^2} - \overline{uv^2}$ $\Leftrightarrow 100c + 10b + a - 100a - 10b - c = (\overline{yx} - \overline{xy})(\overline{yx} + \overline{xy}) +$ $+ (\overline{vu} - \overline{uv})(\overline{vu} + \overline{uv})$ $\Rightarrow 99(c - a) = (10y + x - 10x - y)(10y + x + 10x + y) + (10v + u -$ $- 10u - v)(10v + u + 10u + v)$ $\Rightarrow 99(c - a) = (9y - 9x)(11x + 11y) + (9v - 9u)(11u + 11v)$ $\Rightarrow 99(c - a) = 99(y - x)(y + x) + 99(v - u)(v + u)$ $\Rightarrow c - a = y^2 - x^2 + v^2 - u^2 \quad (*)$ $\text{Din subpunctul a)} \Rightarrow c + a = x^2 + y^2 + u^2 + v^2 \quad (*)$ $\xrightarrow{+}$ $2c = 2y^2 + 2v^2 : 2$ $c = y^2 + v^2$ $\text{Scădem cele două relații (*)} \Rightarrow 2a = 2x^2 + 2u^2 : 2$ $a = x^2 + u^2$ $\text{Iar, din subpunctul a)} \Rightarrow b = 2xy + 2uv$ $b = 2(xy + uv) \leq 9$ $\Rightarrow xy + uv \leq 4$ $a < c \Rightarrow x^2 + u^2 < y^2 + v^2$ $x, y, u, v = \text{cifre nenule} \Rightarrow \begin{cases} x = u = 1 \\ y = 1; v = 2 \end{cases} \text{ sau } \begin{cases} x = 1 = u \\ y = 2; v = 1 \end{cases}$ $\Rightarrow a = 1^2 + 1^2 = 2, c = 1^2 + 2^2 = 5, b = 2(1 \cdot 1 + 1 \cdot 2) = 6$ $\Rightarrow \overline{abc} = 265$ $x, y, u, v = \text{cifre nenule} \Rightarrow \begin{cases} x = u = 1 \\ y = v = 2 \end{cases}$ $\Rightarrow a = 1^2 + 1^2 = 2, c = 2^2 + 2^2 = 8, b = 2(1 \cdot 2 + 1 \cdot 2) = 8$ $\Rightarrow \overline{abc} = 288$	<p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p>

4.	a)	<p>Construim $B'D \parallel C'B, D \in BC$</p> <p>$AB' \perp BC' \Rightarrow AB' \perp B'D \Rightarrow \sphericalangle AB'D = 90^\circ$</p> <p>$BDB'C'$ - paralelogram $\Rightarrow B'D = BC'$, dar $BC' \equiv AB' \Rightarrow B'D = AB'$</p> <p>$\Rightarrow \triangle AB'D$ isoscel și $BD = B'C' = a \Rightarrow BD = AB = a$</p> <p>$\Rightarrow \triangle ABD$ isoscel</p> <p>Cum $\sphericalangle ABD = 120^\circ \Rightarrow \sphericalangle BAD = 30^\circ \Rightarrow \sphericalangle CAD = 90^\circ$</p> <p>$\Rightarrow \triangle CAD$ dreptunghic $\xRightarrow{T.P.} AD = a\sqrt{3}$</p> <p>$\triangle AB'D$ isoscel $\Rightarrow AB' = \frac{a\sqrt{6}}{2}$</p> <p>$\triangle ABB': TP \Rightarrow BB' = \frac{a\sqrt{2}}{2}$</p>	2p
	b)	<p>$(BMC') \cap (ABC) = BM$</p> <p>BM – mediană în $\triangle ABC$ echilateral $\Rightarrow BM \perp AC$</p> <p>Dar $BM \perp (CC'M) \Rightarrow BM \perp C'M$</p> <p>$AC \cap C'M = \{M\}$</p> <p>$AC \subset (ABC), C'M \subset (BMC')$</p> <p>$\Rightarrow \sphericalangle((BMC'), (ABC)) = \sphericalangle C'MC$</p> <p>$tg(\sphericalangle C'MC) = \frac{CC'}{CM} = \frac{\frac{a\sqrt{2}}{2}}{\frac{a}{2}} = \sqrt{2}$</p>	<p>2p</p> <p>1p</p> <p>2p</p> <p>3p</p> <p>3p</p>