Barem de corectare OLM 2025 Clasa a XI-a

P1 – prof. Doru Isac

a) $a_{n+1}-a_n=\sqrt[3]{6+a_n}-a_n=\dfrac{6+a_n-a_n^3}{\sqrt[3]{\left(6+a_n\right)^2}+a_n\sqrt[3]{6+a_n}+a_n^2}=\dfrac{(2-a_n)\left(a_n^2+2a_n+3\right)}{\sqrt[3]{\left(6+a_n\right)^2}+a_n\sqrt[3]{6+a_n}+a_n^2}$	2p
Folosind metoda inducției matematice se arată că $a_n\in(0,2),\forall n\in N^*$	1p
$a_{n+1}-a_n>0,\forall n\in N^*\Rightarrow\left(a_n\right)_{n\geq1}-$ convergent	1p
b) Șirul $\left(a_n\right)_{n\geq1}-$convergent $\Rightarrow(\exists)l\in R$ astfel încât $\lim\limits_{n\to\infty}\left(a_n\right)=l$	2p
Trecând la limită în relația de recurență obținem $l^3-l-6=0\Rightarrow l=2$	1p

P2 - (GM09/2024 – S:L.24.225)

$\sum\limits_{k=1}^{n}\sin^4 x_k+\sum\limits_{k=1}^{n}\cos^4 x_k=\sum\limits_{k=1}^{n}\left(\sin^2 x_k+\cos^2 x_k\right)^2-2\sum\limits_{k=1}^{n}\sin^2 x_k\cdot\cos^2 x_k$	2p
$=n-\dfrac{1}{2}\sum\limits_{k=1}^{n}\sin^2\left(2x_k\right)$	2p
$n-\dfrac{n}{2}<n-\dfrac{1}{2}\sum\limits_{k=1}^{n}\sin^2\left(2x_k\right)<n,\forall n\in N^*$	2p
$\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\sin^4 x_k+\sum\limits_{k=1}^{n}\cos^4 x_k\right)=\infty$	1p

P3

a) Din relația Hamilton-Cayley avem: $A^2-A+I_2=O_2\Rightarrow A^2=A-I_2$	1p
$A^3=A^2-A=A-I_2-A=-I_2$	1p
$A^{2025}=\left(A^3\right)^{675}=-I_2$	1p
b) Se observă că matricea A este inversabilă având inversa $A^{-1}=-A^2$	2p
$X=\left(A^{-1}\right)^2=\left(-A^2\right)^2=A^4=-A$	2p

P4

$\det(A+xB)=\det(A)+c_1x+c_2x^2+\det(B)x^3,\ \ c_1,c_2\in C$	2p
$\det(A+B)=\det(A)+c_1+c_2+\det(B)$ $\det(A+\varepsilon B)=\det(A)+\varepsilon c_1+\varepsilon^2c_2+\varepsilon^3\det(B)$ $\det(A+\varepsilon^2B)=\det(A)+\varepsilon^2c_1+\varepsilon^4c_2+\varepsilon^6\det(B)$	3p
$\varepsilon^3=1$ și $\varepsilon^2+\varepsilon+1=0$	1p
$\det(A+B)+\det(A+\varepsilon B)+\det(A+\varepsilon^2B)=3\left(\det(A)+\det(B)\right)$	1p