OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

Etapa locală - 7 februarie 2025

Clasa a VI-a - Barem

1.

a. $2025=3^4\cdot 5^2$; $N=(4+1)\cdot(2+1)=15$ 1p

Fie $x\in \mathbf{N}$ cel mai mic numar din cele 15 numere. $x+(x+1)+...+(x+14)=2025$ 1p

$15\cdot x+\frac{14\cdot 15}{2}=2025\Rightarrow x=128\Rightarrow 2025=128+129+...+142$ 1p

b. Fie $d=(a;b)\Rightarrow a=d\cdot p;\ b=d\cdot q$; cu $(p;q)=1$. Știind că $a\cdot b=(a;b)\cdot[a;b]$ avem

$3\cdot\frac{d\cdot p\cdot d\cdot q}{d}+4\cdot d=2025\Rightarrow 3\cdot d\cdot p\cdot q+4\cdot d=2025$. 1p

Cum $2025 ⋮ 3$; $3\cdot d\cdot p\cdot q ⋮ 3\Rightarrow 4d ⋮ 3$; $(4;3)=1\Rightarrow d ⋮ 3\Rightarrow d=3\cdot x$; $x\in N\Rightarrow 3\cdot 3\cdot x\cdot p\cdot q+3\cdot 4x=2025|:3\Rightarrow 3\cdot x\cdot p\cdot q+4\cdot x=675$......... 1p

După repetarea raționamentului de trei ori ($x=3\cdot y$; $y=3\cdot z$; $z=3\cdot t$) obținem

$3\cdot t\cdot p\cdot q+4\cdot t=25\Rightarrow t\cdot(3\cdot p\cdot q+4)=25\Rightarrow t=1$; $p\cdot q=7$........ 1p

$d=3^4\cdot t=81\Rightarrow a=81;\ b=567$ 1p

2.

$\sphericalangle AOB=\frac{4}{3}\cdot\sphericalangle DOC$, $\sphericalangle BOC=\frac{9}{4}\cdot\sphericalangle AOB=\frac{9}{4}\cdot\frac{4}{3}\cdot\sphericalangle DOC=3\cdot\sphericalangle DOC$ 1p

$\sphericalangle DOA=\frac{8}{9}\cdot\sphericalangle BOC=\frac{8}{9}\cdot 3\cdot\sphericalangle DOC=\frac{8}{3}\sphericalangle DOC$ 1p

$\sphericalangle AOB+\sphericalangle BOC+\sphericalangle COD+\sphericalangle DOA=360\Rightarrow\frac{4}{3}\cdot\sphericalangle DOC+3\cdot\sphericalangle DOC+\sphericalangle DOC+\frac{8}{3}\cdot\sphericalangle DOC=360\Rightarrow$

$\sphericalangle DOC=45^o$; $\sphericalangle AOB=60^o$; $\sphericalangle BOC=135^o$; $\sphericalangle DOA=120^o$ 2p

OE bisectoarea $\sphericalangle AOB\Rightarrow\sphericalangle AOE=\sphericalangle EOB=\frac{\sphericalangle AOB}{2}=\frac{60^o}{2}=30^o$

OF bissectoarea $\sphericalangle AOD\Rightarrow\sphericalangle AOF=\sphericalangle FOD=\frac{\sphericalangle AOD}{2}=\frac{120^o}{2}=60^o$

$\sphericalangle EOF=\sphericalangle EOA+\sphericalangle AOF=90^o\Rightarrow OE\perp OF$ 2p

OG bisectoarea $\sphericalangle BOC\Rightarrow\sphericalangle BOG=\sphericalangle GOC=\frac{\sphericalangle BOC}{2}=\frac{135^o}{2}=67^o30'$

$\sphericalangle FOG=\sphericalangle FOD+\sphericalangle DOC+\sphericalangle COG=60^o+45^o+67^o30'=172^030'$ 1p

3.

$n=2k+1;\ k\in\mathbf{N}\Rightarrow n^4-5n-20$ par $\Rightarrow p$ par 2p

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Deci $p=2;\ n=3$ 3p

4.

$A_oA_{2025}=A_oA_1+A_1A_2+...+A_{2024}A_{2025}=1+2+...+2025=\frac{2025\cdot 2026}{2}=1013\cdot 2025$ 2p

$A_0M=\frac{A_0A_{2025}}{2}=\frac{2025\cdot 1013}{2}$ 1p

$M\in A_kA_{k+1}\Leftrightarrow A_0A_k<A_0M<A_0A_{k+1}$ 2p

$\frac{k\cdot(k+1)}{2}<\frac{2025\cdot 1013}{2}<\frac{(k+1)(k+2)}{2}$ 1p

$(31\cdot 45)^2<1431\cdot 1432<45^2\cdot 1013<1432\cdot 1432\Rightarrow k=1431$ 1p

NOTĂ Orice soluție corectă se punctează similar baremului