OLIMPIADA NAŢIONALĂ DE MATEMATICĂ
Etapa locală-17.02.2018
Clasa a XI-a
Bareme de corectare si de notare

1.

a) $\alpha = 1, \beta = -3, \gamma = 3, \delta = -1$…………………………………..2p

b) $A^{-1} = \begin{pmatrix} 1 & -5 & 19 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix}$ ………………………………………2p

c) $A^n = \begin{pmatrix} 1 & 5n & \frac{n(25n-13)}{2} \\ 0 & 1 & 5n \\ 0 & 0 & 1 \end{pmatrix}$ ..…………………………………..3p

2.

a) Fie $k \in \{1, 2, 3, \dots, 10\}$.
In I_{10}suntC_{10}^k minori nenuli de ordinul k…………………………2p
$m(I_{10}) = C_{10}^1 + C_{10}^2 + \cdots + C_{10}^{10} = 2^{10} - 1$ ……………………1p

b) $detA \neq 0 \Rightarrow rang\ A = 10$
Fie $k \in \{1, 2, 3, \dots, 10\}$.
In A gasim cel putin un minor nenul de ordin$k$ la fiecare alegere a k coloane, deoarece in caz contrar am obtine ca printre coloanele selectate cel putin una este combinatie liniara de coloanele din A , adica$detA = 0$, fals……...2p
A arecelputinC_{10}^kminorinenuli de ordinul$k$…………………..…1p
$m(A) \geq C_{10}^1 + C_{10}^2 + \cdots + C_{10}^{10} = 2^{10} - 1$ ………………………...1p

3.

a) Monotonia…………………………………………………..2p

Mărginirea…………………………………………………...2p

b) $\lim_{n\to\infty} \left(\frac{4arctg\ n - \pi}{\pi}\right)^n = e^{-\frac{4}{\pi}}$ …………..………………….3p

4.

$L = \lim_{x\to\infty} x\left(\sqrt{a + \frac{b}{x} + \frac{c}{x^2}} + \sqrt{a' + \frac{b'}{x} + \frac{c'}{x^2}} - \alpha - \frac{\beta}{x}\right) \in \mathbb{R}^* \Rightarrow \sqrt{a} + \sqrt{a'} = \alpha$…3p

$L = \lim_{x\to\infty} \left(\sqrt{ax^2 + bx + c} + \sqrt{a'x^2 + b'x + c'} - \sqrt{a}x - \sqrt{a'}x\right) - \beta = \frac{b}{2\sqrt{a}} + \frac{b'}{2\sqrt{a'}} - \beta \in \mathbb{R}^*$….3p

$\beta \in \mathbb{R} - \left\{\frac{b}{2\sqrt{a}} + \frac{b'}{2\sqrt{a'}}\right\}$………………………………………………………..1p