Olimpiada de Matematică –etapa locală- Galaţi

17 februarie 2018

Clasa a VII-a

Barem de evaluare

- **Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul maxim corespunzător.**
- **Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.**

Nr. problemei	Soluţie, rezolvare	Punctaj
1.	*a*) Se fac calcule.	2p
	b) Conform punctului *a*): $\frac{2^2}{1\cdot 3}=\frac{2^2}{2^2-1}=\frac{1}{1}+\frac{1}{3}$ $\frac{4^2}{3\cdot 5}=\frac{2}{3}+\frac{2}{5}$ $\frac{6^2}{5\cdot 7}=\frac{3}{5}+\frac{3}{7}$ $\frac{8^2}{7\cdot 9}=\frac{4}{7}+\frac{4}{9}$ . . . $\frac{2018^2}{2017\cdot 2019}=\frac{1009}{2017}+\frac{1009}{2019}$	1p
	Însumând aceste egalităţi, obţinem: $A=\frac{1}{1}+\frac{1}{3}+\frac{2}{3}+\frac{2}{5}+\frac{3}{5}+\frac{3}{7}+\frac{4}{7}+\frac{4}{9}+...+\frac{1009}{2017}+\frac{1009}{2019}=$	1p
	$1+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{2}{5}+\frac{3}{5}\right)+\left(\frac{3}{7}+\frac{4}{7}\right)+...+\left(\frac{1008}{2017}+\frac{1009}{2017}\right)+\frac{1009}{2019}=$	1p
	$\underbrace{1+1+1+...+1}_{1009 \text{ termeni}}+\frac{1009}{2019}=1009\frac{1009}{2019}\Rightarrow\left\{1009\frac{1009}{2019}\right\}=\frac{1009}{2019}$	1p
	$\frac{1009}{2019}<0,5\Rightarrow\frac{1009}{2019}<\frac{1}{2}\Rightarrow 2018<2019\quad(A).$	1p

2.	Metoda 1. $[BP$ este bisectoarea $\measuredangle ABM \overset{T.B}{\Rightarrow}$ $\dfrac{PM}{MA}=\dfrac{MB}{AB}\Rightarrow\dfrac{PM}{PA}=\dfrac{4}{5}\Rightarrow\dfrac{PM}{MA}=\dfrac{4}{9}$; Fie $BE \perp AM$, $E \in AM$ $\dfrac{A_{\triangle BMP}}{A_{ABM}}=\dfrac{\frac{PM\cdot BE}{2}}{\frac{AM\cdot BE}{2}}\Rightarrow\dfrac{A_{\triangle BMP}}{A_{ABM}}=\dfrac{PM}{MA}=\dfrac{4}{9}$; $[AM]-$ mediană în $\triangle ABC\Rightarrow A_{\triangle ABM}=\dfrac{1}{2}\cdot A_{\triangle ABC}$; $\dfrac{A_{\triangle BMP}}{A_{ABC}}=\dfrac{A_{\triangle BMP}}{2\cdot A_{\triangle ABM}}=\dfrac{1}{2}\cdot\dfrac{4}{9}=\dfrac{2}{9}$. Metoda 2. Fie $BP\cap AC=\{N\}$. $[BN$ este bisectoarea $\measuredangle ABC\overset{T.B}{\Rightarrow}\dfrac{AN}{NC}=\dfrac{AB}{BC}\Rightarrow\dfrac{AN}{NC}=\dfrac{5}{8}\Rightarrow\dfrac{AN}{AC}=\dfrac{5}{13}$; $\left.\begin{array}{l}\triangle BCN\\ \text{transversala A-P-M}\end{array}\right\}\overset{T.M}{\Rightarrow}\dfrac{BP}{PN}\cdot\dfrac{AN}{AC}\cdot\dfrac{MC}{MB}=1\Leftrightarrow\dfrac{BP}{PN}\cdot\dfrac{5}{13}\cdot1=1\Leftrightarrow\dfrac{BP}{PN}=\dfrac{13}{5}$; $\dfrac{A_{\triangle BPM}}{A_{\triangle BCN}}=\dfrac{BP}{BN}\cdot\dfrac{BM}{BC}=\dfrac{13}{18}\cdot\dfrac{1}{2}=\dfrac{13}{36}$; $\dfrac{A_{\triangle BNC}}{A_{\triangle ABC}}=\dfrac{CN}{AC}=\dfrac{8}{13}$; $\dfrac{A_{\triangle BPM}}{A_{\triangle ABC}}=\dfrac{13}{36}\cdot\dfrac{8}{13}=\dfrac{2}{9}$.	3p 2p 2p

3.	$\dfrac{1}{x_2+x_3+\ldots+x_n}+\dfrac{2}{x_1+x_3+\ldots+x_n}+\ldots+\dfrac{n}{x_1+x_2+x_3+\ldots+x_{n-1}}=b$	
	Înmulţind egalitatea cu a, se obţine:	
	$\dfrac{a}{x_2+x_3+\ldots+x_n}+\dfrac{2\cdot a}{x_1+x_3+\ldots+x_n}+\ldots+\dfrac{n\cdot a}{x_1+x_2+x_3+\ldots+x_{n-1}}=a\cdot b\Leftrightarrow$	1p
	$\dfrac{x_1+x_2+x_3+\ldots+x_n}{x_2+x_3+\ldots+x_n}+\dfrac{2\cdot\left(x_1+x_2+x_3+\ldots+x_n\right)}{x_1+x_3+\ldots+x_n}+\ldots+\dfrac{n\cdot\left(x_1+x_2+x_3+\ldots+x_n\right)}{x_1+x_2+x_3+\ldots+x_{n-1}}$	1p
	Dar $\dfrac{x_1+x_2+x_3+\ldots+x_n}{x_2+x_3+\ldots+x_n}=\dfrac{x_1}{x_2+x_3+\ldots+x_n}+1;$	3p
	$\dfrac{2\cdot\left(x_1+x_2+x_3+\ldots+x_n\right)}{x_1+x_3+\ldots+x_n}=\dfrac{2\cdot x_2}{x_1+x_3+\ldots+x_n}+2;$	
	$\dfrac{3\cdot\left(x_1+x_2+x_3+\ldots+x_n\right)}{x_1+x_2+x_4+\ldots+x_n}=\dfrac{3\cdot x_3}{x_1+x_2+x_4+\ldots+x_n}+3;$	
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	$\dfrac{n\cdot\left(x_1+x_2+x_3+\ldots+x_n\right)}{x_1+x_2+x_3+\ldots+x_{n-1}}=\dfrac{n\cdot x_n}{x_1+x_2+x_3+\ldots+x_{n-1}}+n$	
	Însumând aceste egalităţi se obţine:	
	S+1+2+3+...+n $=a\cdot b\Rightarrow S=ab-\dfrac{n\cdot\left(n+1\right)}{2}$.	2p

4.	a) Metoda 1. Fie $AB=l$; $BE=\frac{1}{3}\cdot BO=\frac{1}{6}\cdot BD=\frac{1}{5}\cdot ED\Rightarrow\frac{BE}{DE}=\frac{1}{5}$, $DF=\frac{1}{2}\cdot DO=\frac{1}{4}\cdot BD=\frac{1}{3}\cdot FB\Rightarrow\frac{DF}{BF}=\frac{1}{3}$	1p
	$BM \parallel AD \overset{T.F.A}{\Rightarrow} \triangle BME \sim \triangle DAE \overset{def.}{\Rightarrow} \frac{BM}{AD}=\frac{BE}{DE}=\frac{ME}{AE}\Rightarrow$ $\frac{BM}{l}=\frac{1}{5}\Rightarrow BM=\frac{1}{5}\cdot l\Rightarrow MC=\frac{4}{5}\cdot l$;	1p
	$DN \parallel AB \overset{T.F.A}{\Rightarrow} \triangle DNF \sim \triangle BAF \overset{def.}{\Rightarrow} \frac{DN}{AB}=\frac{NF}{AF}=\frac{DF}{BF}\Rightarrow$ $\frac{DN}{l}=\frac{1}{3}\Rightarrow DN=\frac{1}{3}\cdot l\Rightarrow NC=\frac{2}{3}\cdot l$,	1p
	$A_{\triangle AMN}=A_{ABCD}-A_{\triangle ADN}-A_{\triangle CNM}-A_{\triangle AMB}=$ $l^2-\frac{l^2}{6}-\frac{4\cdot l^2}{15}-\frac{l^2}{10}=\frac{7\cdot l^2}{15}$;	1p
	$p\%\cdot A_{ABCD}=A_{\triangle AMN}\Rightarrow p=\frac{7}{15}=46,(6)$.	1p
	Metoda 2. Fie $AB=l$. Fie $\triangle$COB cu transversala A-E-M $\overset{T.Menelaus}{\Rightarrow}$ $\frac{BE}{EO}\cdot\frac{OA}{CA}\cdot\frac{CM}{MB}=1\Rightarrow\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{CM}{MB}=1\Rightarrow CM=4\cdot MB\Rightarrow$ $CM=\frac{2\cdot l}{3}$, Fie $\triangle$COD cu transversala A-F-N $\overset{T.Menelaus}{\Rightarrow}$ $\frac{OF}{FD}\cdot\frac{DN}{NC}\cdot\frac{CA}{OA}=1\Rightarrow 1\cdot\frac{DN}{NC}\cdot 2=1\Rightarrow CN=2\cdot DN\Rightarrow$ $DN=\frac{l}{3}$ $A_{\triangle AMN}=A_{ABCD}-A_{\triangle ADN}-A_{\triangle CNM}-A_{\triangle AMB}=$ $l^2-\frac{l^2}{6}-\frac{4\cdot l^2}{15}-\frac{l^2}{10}=\frac{7\cdot l^2}{15}$; $p\%\cdot A_{ABCD}=A_{\triangle AMN}\Rightarrow p=\frac{7}{15}=46,(6)$.	
	$b)$În $\triangle$CDB: M $\in$ BC, N $\in$ DC, I $\in$ BD, $\frac{DN}{NC}=\frac{1}{2},\ \frac{CM}{MB}=4,\ \frac{IB}{ID}=\frac{1}{2}\Rightarrow$	1p
	$\frac{DN}{NC}\cdot\frac{CM}{MB}\cdot\frac{IB}{ID}=1 \overset{R.T.M.}{\Rightarrow} M, N, I$ coliniare.	1p