BAREM DE CORECTARE ȘI NOTARE

Clasa a V-a

1. **a) (3p)** Arătați că numărul $x = \left(3^{1+2+3+\ldots+42} + 2\cdot 3^{1+3+5+\ldots+59}\right) : 29$ este cub perfect.

b) (4p) Arătați că numărul x poate fi scris ca o sumă de patru pătrate perfecte nenule.

Prof. Sascău Gabriela

<u>Soluție</u>:

a) $1 + 2 + 3 + \ldots + 42 = 42 \cdot 43 : 2 = 903$

$1 + 3 + 5 + \ldots + 59 = 1 + 2 + 3 + \ldots + 59 - (2 + 4 + \ldots + 58) = 59 \cdot 60 : 2 - 2 \cdot 29 \cdot 30 : 2 = 30^2 = 900$

$x = (3^{903} + 2\cdot 3^{900}) : 29 = 3^{900}(27 + 2) : 29 = 3^{900} = (3^{300})^3$

b) $x = 3^{900} = 3^{898} \cdot 3^2 = 3^{898} \cdot 9$. Dar 9 nu se poate scrie ca sumă a patru pătrate perfecte nenule.

$x = 3^{896}\cdot 3^4 = 3^{896}\cdot 81 = 3^{896}\cdot(4 + 16 + 25 + 36) = (3^{448}\cdot 2)^2 + (3^{448}\cdot 4)^2 + (3^{448}\cdot 5)^2 + (3^{448}\cdot 6)^2$

<u>Barem</u>

a) $3^{1+2+3+\ldots+42} = 3^{903}$; $3^{1+3+5+\ldots+59} = 3^{900}$	1p
$x = 3^{900}(27 + 2) : 29 = 3^{900}$	1p
$x = (3^{300})^3$	1p
b) $x = 3^{900} = 3^{896} \cdot 3^4 = 3^{896} \cdot 81 = 3^{896} \cdot (4 + 16 + 25 + 36)$	2p
$x = 3^{896} \cdot 2^2 + 3^{896} \cdot 4^2 + 3^{896} \cdot 5^2 + 3^{896} \cdot 6^2 = (3^{448}\cdot 2)^2 + (3^{448}\cdot 4)^2 + (3^{448}\cdot 5)^2 + (3^{448}\cdot 6)^2$	2p

2. Se consideră mulţimile $A = \{5n+4; 5n+5; 5n+9 / n \in \mathbb{N}\}$ şi $B = \{m^2 + 2017 / m \in \mathbb{N}\}$.

a) **(2p)** Stabiliţi dacă $2017 \in A$.

b) **(5p)** Calculaţi $A \cap B$.

Prof. Schroder Laura

<u>Soluţie</u>:

a) $2017 = 5 \cdot 403 + 2$ deci $2017 \notin$ A.

b) $u(m^2) \in \{0,1,4,5,6,9\}$ prin urmare $u(m^2 + 2017) \in \{1,2,3,6,7,8\}$.

$u(5\cdot n) \in \{0,5\}$ prin urmare $u(A) \in \{0,4,5,9\}$, deci $A \cap B = \varnothing$.

<u>Barem</u>

a)	$2017 = 5\cdot 403 + 2$	1p
	$2017 \notin$ A	1p
b)	$u(m^2) \in \{0,1,4,5,6,9\}$	1p

	$u\left(m^2+2017\right)\in\{1,2,3,6,7,8\}$	1p
	$u(5\cdot n)\in\{0,5\}$	1p
	$u(A)\in\{0,4,5,9\}$	1p
	$A\cap B=\varnothing$	1p

3. a) (3p) Arătaţi că $\underbrace{\overline{abab...ab}}_{2016\ litere}+\underbrace{\overline{baba...ba}}_{2016\ litere}=\underbrace{\overline{aa...a}}_{de\ 2016\ ori}+\underbrace{\overline{bb...b}}_{de\ 2016\ ori}$.

b) (4p) Demonstraţi că $37\,/\left(\underbrace{\overline{abab...ab}}_{2016\ litere}+\underbrace{\overline{baba...ba}}_{2016\ litere}\right)$.

Prof. Ciobîcă Constantin

***Soluţie**:*

a)

$$\underbrace{\overline{abab...ab}}_{2016\ litere}+\underbrace{\overline{baba...ba}}_{2016\ litere}=a\cdot1\underbrace{00...0}_{de\ 2015\ ori}+b\cdot1\underbrace{00...0}_{de\ 2014\ ori}+a\cdot1\underbrace{00...0}_{de\ 2013\ ori}+b\cdot1\underbrace{00...0}_{de\ 2012\ ori}+...+a\cdot10+b+$$

$$+b\cdot1\underbrace{00...0}_{de\ 2015\ ori}+a\cdot1\underbrace{00...0}_{de\ 2014\ ori}+b\cdot1\underbrace{00...0}_{de\ 2013\ ori}+a\cdot1\underbrace{00...0}_{de\ 2012\ ori}+...+b\cdot10+a=$$

$$=a\cdot1\underbrace{00...0}_{de\ 2015\ ori}+a\cdot1\underbrace{00...0}_{de\ 2014\ ori}+...+a\cdot10+a+b\cdot1\underbrace{00...0}_{de\ 2015\ ori}+b\cdot1\underbrace{00...0}_{de\ 2014\ ori}+...+b\cdot10+b=$$

$$=\underbrace{\overline{aa...a}}_{de\ 2016\ ori}+\underbrace{\overline{bb...b}}_{de\ 2016\ ori}.$$

b) $\underbrace{\overline{aa...a}}_{de\ 2016\ ori}+\underbrace{\overline{bb...b}}_{de\ 2016\ ori}=a\cdot\underbrace{\overline{11...1}}_{de\ 2016\ ori}+b\cdot\underbrace{\overline{11...1}}_{de\ 2016\ ori}=\underbrace{\overline{11...1}}_{de\ 2016\ ori}\cdot(a+b)=111\cdot\underbrace{1001001...1001}_{2014\ cifre}\cdot(a+b)=$

$=37\cdot3\cdot\underbrace{1001001...1001}_{2014\ cifre}\cdot(a+b)$

Barem

a)	$\underbrace{\overline{abab...ab}}_{2016\ litere}=a\cdot1\underbrace{00...0}_{de\ 2015\ ori}+b\cdot1\underbrace{00...0}_{de\ 2014\ ori}+a\cdot1\underbrace{00...0}_{de\ 2013\ ori}+b\cdot1\underbrace{00...0}_{de\ 2012\ ori}+...+a\cdot10+b$	1p
	$\underbrace{\overline{baba...ba}}_{2016 litere}=b\cdot1\underbrace{00...0}_{de\ 2015\ ori}+a\cdot1\underbrace{00...0}_{de\ 2014\ ori}+b\cdot1\underbrace{00...0}_{de\ 2013\ ori}+a\cdot1\underbrace{00...0}_{de\ 2012\ ori}+...+b\cdot10+a$	1p
	$\underbrace{\overline{abab...ab}}_{2016\ litere}+\underbrace{\overline{baba...ba}}_{2016\ litere}=a\cdot1\underbrace{00...0}_{de\ 2015\ ori}+a\cdot1\underbrace{00...0}_{de\ 2014\ ori}+...+a\cdot10+a+$ $+b\cdot1\underbrace{00...0}_{de\ 2015\ ori}+b\cdot1\underbrace{00...0}_{de\ 2014\ ori}+...+b\cdot10+b=\underbrace{\overline{aa...a}}_{de\ 2016\ ori}+\underbrace{\overline{bb...b}}_{de\ 2016\ ori}$	1p
b)	$\underbrace{\overline{aa...a}}_{de\ 2016\ ori}+\underbrace{\overline{bb...b}}_{de\ 2016\ ori}=a\cdot\underbrace{\overline{11...1}}_{de\ 2016\ ori}+b\cdot\underbrace{\overline{11...1}}_{de\ 2016\ ori}=\underbrace{\overline{11...1}}_{de\ 2016\ ori}\cdot(a+b)$	1p
	$\underbrace{\overline{11...1}}_{de\ 2016\ ori}=111\cdot\underbrace{1001001...1001}_{2014\ cifre}=37\cdot3\cdot\underbrace{1001001...1001}_{2014\ cifre}$	2p
	Finalizare $37\,/\left(\underbrace{\overline{abab...ab}}_{2016\ litere}+\underbrace{\overline{baba...ba}}_{2016\ litere}\right)$	1p

Notă: Orice altă soluţie corectă se va puncta corespunzător.