OLIMPIADA DE MATEMATICĂ
ETAPA LOCALĂ
21 IANUARIE 2017
CLASA a VII-a
Bareme

Subiectul 1.

	$\frac{3}{1\cdot 2}+\frac{4}{2\cdot 3}+\frac{5}{3\cdot 4}+\ldots+\frac{n+2}{n\cdot(n+1)}-\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right)=$ $\left(\frac{3}{1\cdot 2}-\frac{1}{1}\right)+\left(\frac{4}{2\cdot 3}-\frac{1}{2}\right)+\left(\frac{5}{3\cdot 4}-\frac{1}{3}\right)+\ldots+\left(\frac{n+2}{n\cdot(n+1)}-\frac{1}{n}\right)=$	**3p**
	$=\frac{3-2}{1\cdot 2}+\frac{4-3}{2\cdot 3}+\frac{5-4}{3\cdot 4}+\ldots+\frac{n+2-n}{n\cdot(n+1)}=$ $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n\cdot(n+1)}=$ $\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n}-\frac{1}{n+1}=\frac{1}{1}-\frac{1}{n+1}$	**3p**
	$1-\frac{1}{n+1}<1$	**1p**

Subiectul 2.

	$\sqrt{3}>\sqrt{2}\Rightarrow\left\|\sqrt{2}-\sqrt{3}\right\|=\sqrt{3}-\sqrt{2}$ $\sqrt{2(a+1)^2}-2\sqrt{2}=\|b+2\|\cdot\sqrt{3}-\left\|\sqrt{2}-\sqrt{3}\right\|$ $\Leftrightarrow\|a+1\|\sqrt{2}-2\sqrt{2}=\|b+2\|\sqrt{3}-\sqrt{3}+\sqrt{2}\Leftrightarrow$	**3p**
	$\sqrt{2}\left(\|a+1\|-3\right)=\sqrt{3}\left(\|b+2\|-1\right)$	**1p**
	Cum a şi b sunt numere raţionale $\Rightarrow\|a+1\|-3=\|b+2\|-1=0$ Se obţin soluţiile $(a,b)\in\{(2,-1),(2,-3),(-4,-1),(-4,-3)\}$	**3p**

Subiectul 3.

a.	$A_{BMC}=\frac{BC\cdot BM}{2}=\frac{l\cdot BM}{2}$, unde am notat cu l lungimea laturii pătratului $A_{DMC}=\frac{DC\cdot MN}{2}=\frac{l\cdot l}{2}$, unde am notat cu N piciorul perpendicularei din M pe DC $BM<l\Rightarrow A_{BMC}<A_{DMC}$	**3p**
b.	$A_{BMC}=\frac{CM\cdot BE}{2}=\frac{l\cdot BM}{2}\Rightarrow CM=\frac{MB\cdot l}{18}$ $A_{DMC}=\frac{CM\cdot 24}{2}=\frac{l\cdot l}{2}\Rightarrow CM=\frac{l\cdot l}{24}$ $\frac{MB\cdot l}{18}=\frac{l\cdot l}{24}\Rightarrow MB=\frac{l\cdot l\cdot 18}{l\cdot 24}=\frac{3}{4}l=\frac{3}{4}AB$	**4p**

Subiectul 4.

	$MB \perp QN, AD \perp QN, PC \perp QN \Rightarrow MB \parallel AD \parallel PC$	**2p**
	Aplicând Teorema lui Thales în ΔOPC, $AD \parallel PC \Rightarrow \frac{OA}{AP} = \frac{OD}{DC}$	**2p**
	Aplicând Teorema lui Thales în ΔOBM, $AD \parallel MB \Rightarrow \frac{OA}{AM} = \frac{OD}{DB}$	**2p**
	Cum $AP = AM \Rightarrow \frac{OA}{AP} = \frac{OA}{AM} \Rightarrow \frac{OD}{DC} = \frac{OD}{DB} \Rightarrow DC = DB$	