MINISTERUL EDUCAŢIEI NAŢIONALE

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BAREME OLIMPIADA DE MATEMATICA

FAZA LOCALA- 26 FEBRUARIE 2017
CLASA a VIII-a

1.

a) Daca x=$2\sqrt{7}-3\sqrt{3}$ si y=$|-2\sqrt{7}-3\sqrt{3}|$, calculati x^2+y^2.

b) Calculati : $(\sqrt{3}+2)^{-1}+\sqrt{(1-\sqrt{3})^2}$.

c)Aratati ca nu exista patrat perfect de forma 4m+3, oricare ar fi m numar natural

Solutie :

a) $x^2+y^2=110$ **2p**

b) $\frac{1}{2+\sqrt{3}}+I1-\sqrt{3}\,I=1$ **2p**

c) $(2p)^2=4p^2$, $(2p+1)^2=4p^2+4p+1=4(p^2+p)+1$.Restul impartirii unui patrat perfect la 4 este 0 sau 1 **3p**

2.Pentru orice numar natural n$\geq$ 2 notam

$$S_n=\frac{1}{n^2+1}+\frac{1}{n^2+2}+\frac{1}{n^2+3}+\cdots+\frac{1}{n^2+n}\ .$$

a) Aratati ca $\frac{1}{n+1}<S_n<\frac{1}{n}$, pentru orice n$\geq$ 2 .

b) Calculati partea intreaga a numarului: $\frac{1}{2}\cdot\left(\frac{1}{S_{20}}+\frac{1}{S_{16}}\right)$.

Solutie : a) $\frac{1}{n+1}=\frac{n}{n^2+n}<S_n<\frac{n}{n^2+1}<\frac{1}{n}$ **4 p**

b) $n<\frac{1}{S_n}<n+1$

$\rightarrow 20<\frac{1}{S_{20}}<21\ si\ 16<\frac{1}{S_{16}}<17\rightarrow 36<\frac{1}{S_{20}}+\frac{1}{S_{16}}<38\rightarrow 18<\frac{1}{2}\cdot\left(\frac{1}{S_{20}}+\frac{1}{S_{16}}\right)<19\rightarrow\left[\frac{1}{2}\cdot\left(\frac{1}{S_{20}}\right.\right.$ **3 p**

=18

3.Patratul ABCD si dreptunghiul AEFD sunt in plane diferite cu DE $\perp$ AB .Stiind ca CD=1, ED=$\sqrt{3}$ să se calculeze :

a) d(E,FC);
b) d(B,DE);

Solutie : a) $AB \perp AD$ si $AB \perp DE \rightarrow AB \perp (AEFD) \rightarrow CD \perp (AEFD) \rightarrow CD \perp EF$ si $EF \perp DF \rightarrow EF \perp (CDF) \rightarrow EF \perp CF \rightarrow d(E,FC)=EF=1$ **4p**

b)$AE=\sqrt{2}$ si $AP \perp DE$, $AP=\frac{\sqrt{2}}{\sqrt{3}}$, din T.3.P. avem $BP=d(B,DE)=\frac{\sqrt{15}}{3}$ **3 p**

4.Pe planul triunghiului echilateral ABC se duc , in varfuri , perpendicularele AA_1 , BB_1 , CC_1 incat A_1 , B_1 sa fie de o parte a planului (ABC) , iar C_1 de partea opusa , $AA_1=3a$, $BB_1=CC_1=AB=a$, se cere :
a) aratati ca $MN \perp AC$, unde $BC \cap B_1C_1=\{M\}$, $AC \cap A_1C_1=\{N\}$.
b) aratati ca $MN \perp (ACA_1)$.
c) calculati $d(A, (A_1B_1C_1))$

Solutie: a)$\Delta AA_1N \sim \Delta CC_1N \rightarrow AN=\frac{3a}{4}$, $CN=\frac{a}{4}$. $\Delta BB_1M \equiv \Delta CC_1M \rightarrow BM=MC=\frac{a}{2} \rightarrow AM=h=\frac{a\sqrt{3}}{2}$. ΔMNC , $MC=\frac{a}{2}$, $NC=\frac{a}{4}$, $\widehat{C}=60^0$ se deduce ca este tr.dreptunghic in $\widehat{N} \rightarrow MN \perp AC$. **3p**

b)$MN=\frac{a\sqrt{3}}{4}$. $\Delta CMC_1 \rightarrow MC_1=\frac{a\sqrt{5}}{2}$. $\Delta NCC_1 \rightarrow NC_1=\frac{a\sqrt{17}}{4}$. ΔMNC_1 (R.T.P)$\rightarrow$ dreptunghic in $\widehat{N} \rightarrow MN \perp A_1C_1 \rightarrow MN \perp (AA_1C_1)$ **2p**

c)Duc $AP \perp A_1C_1$, $A_1C_1 \perp MN$, $AN \perp MN \rightarrow$ din R.2 T.3.P. avem $AP \perp (A_1B_1C_1) \rightarrow d(A,(A_1B_1C_1))=AP=h=\frac{AA_1 \cdot AN}{A_1N}=\frac{3a\sqrt{17}}{17}$. **2p**