EVALUAREA NAȚIONALA 2023

Rezolvarea subiectelor la matematică

SUBIECTUL I

1.	2.	3.	4.	5.	6.
b	c	a	c	d	b

SUBIECTUL al II-lea

1.	2.	3.	4.	5.	6.
c	c	c	b	b	c

SUBIECTUL al III-lea

Mate.info.ro
profu' de mate

① a) Peste 2 ani, Maria va avea vârsta de $14+2=16$ ani, iar tatăl ei $40+2=42$ ani

$42+16=58$ de ani

$58 \neq 60$

Nu este posibil ca peste 2 ani suma vârstelor lor să fie egală cu 60 de ani.

b) Notez cu x cerința problemei

$$14+x=\frac{40+x}{2} \mid \cdot 2$$

$$28+2x=40+x$$

$$2x-x=40-28$$

$$x=12 \text{ ani}$$

② a) $\frac{1}{(x+1)(x+2)} + \frac{1}{x+2} = \frac{1}{(x+1)(x+2)} +$ (amplificăm cu $x+1$)

$$+\frac{x+1}{(x+1)(x+2)} = \frac{\cancel{x+2}^{1}}{(x+1)(\cancel{x+2}_{1})} = \frac{1}{x+1}$$

b) Folosim rezultatul de la a)

$$E(x) = \frac{1}{x+1} : \frac{x+3}{5(x+1)}$$

$$E(x) = \frac{1}{\cancel{x+1}} \cdot \frac{5\cancel{(x+1)}}{x+3}$$

$$E(x) = \frac{5}{x+3} \Bigg| \Rightarrow \frac{5}{x+3} = \frac{x-3}{8}$$

$$E(x) = \frac{x-3}{8}$$

$$(x+3)(x-3) = 5 \cdot 8$$

$$x^2 - 9 = 40$$

$$x^2 = 49 \Rightarrow x = \pm\sqrt{49}$$

$$x_1 = -7 \; ; \; x_2 = +7 \Bigg| \Rightarrow S = \{-7; 7\}$$

$$x \in \mathbb{R} \setminus \{-3; -2; -1\} \Bigg| \; -7+7 = 0$$

③ $f: \mathbb{R} \to \mathbb{R}, \; f(x) = -x+5$

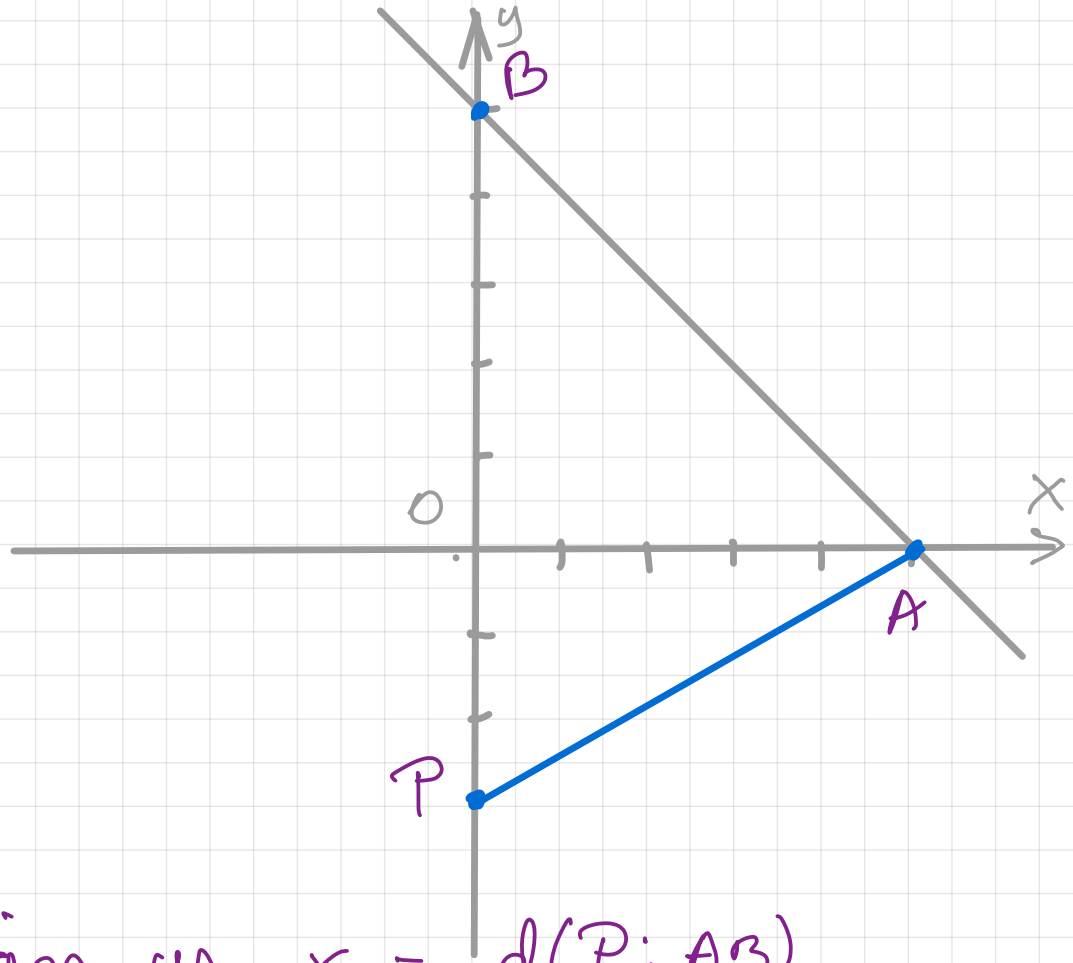
a) $f(4) = -4+5 = 1$

$f(6) = -6+5 = -1$

$f(4) + f(6) = 1 + (-1) = 0$

b) $x = 0$
$f(0) = 5$ $\Big| \Rightarrow G_f \cap OY = B(0;5)$

$f(x) = 0$
$-x + 5 = 0$
$x = 5$ $\Big| \Rightarrow G_f \cap OX = A(5;0)$



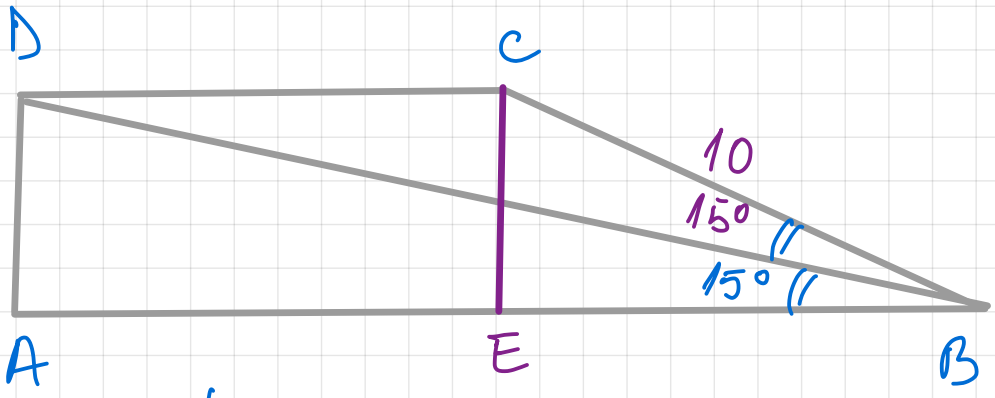
Notăm cu $x = d(P; AB)$

ΔAOB este dreptunghic isoscel ($OA = OB = 5$) $\Rightarrow AB = 5\sqrt{2}$

$$A_{\Delta ABP} = \frac{AO \cdot AP}{2}$$
$$A_{\Delta ABP} = \frac{x \cdot AB}{2}$$
$$\Big| \Rightarrow \frac{5 \cdot 8}{2} = \frac{x \cdot 5\sqrt{2}}{2} \Big| \cdot \frac{2}{5}$$

$\Rightarrow x = \frac{8\sqrt{2}}{\sqrt{2}} \Rightarrow x = 4\sqrt{2}$

(4)



a) BD - bisectoarea $\sphericalangle ABC \Rightarrow$

$\Rightarrow \sphericalangle ABD \equiv \sphericalangle CBD = 15^\circ \Rightarrow \sphericalangle ABC = 30^\circ$

$\sphericalangle ABC + \sphericalangle BCD = 180^\circ$

$30^\circ + \sphericalangle BCD = 180^\circ \Rightarrow \sphericalangle BCD = 150^\circ$

b) $\sphericalangle ABD \equiv \sphericalangle BDC$ (alterne interne) $\Rightarrow \sphericalangle BDC \equiv \sphericalangle CBD \Rightarrow$

$\sphericalangle ABD \equiv \sphericalangle CBD$

$\Rightarrow \Delta DBC$ - isoscel $\Rightarrow DC = BC = 10$ cm

Ducem $CE \perp AB$

$AECD$ - dreptunghi $\Rightarrow AE = DC = 10$ cm

În ΔBEC, dreptunghic, $\sphericalangle CBE = 30^\circ \Rightarrow$

$\Rightarrow CE = \frac{BC}{2} = \frac{10}{2} = 5$ cm.

$CE = AD = 5$ cm.

$CE^2 + BE^2 = BC^2 \Rightarrow BE = 5\sqrt{3}$ cm

$AB = AE + EB$

$AB = (10 + 5\sqrt{3})$ cm

Verificăm dacă $AB - AD < 14$ cm

$10 + 5\sqrt{3} - 5 < 14$

$5 + 5\sqrt{3} < 14 \quad |-5$

$5\sqrt{3} < 9$

$\sqrt{75} < \sqrt{81}$ adevărat!

(5) a) În triunghiul dreptunghic ABC:

$AB^2 + BC^2 = AC^2$

$(9\sqrt{10})^2 + BC^2 = 30^2$

$810 + BC^2 = 900$

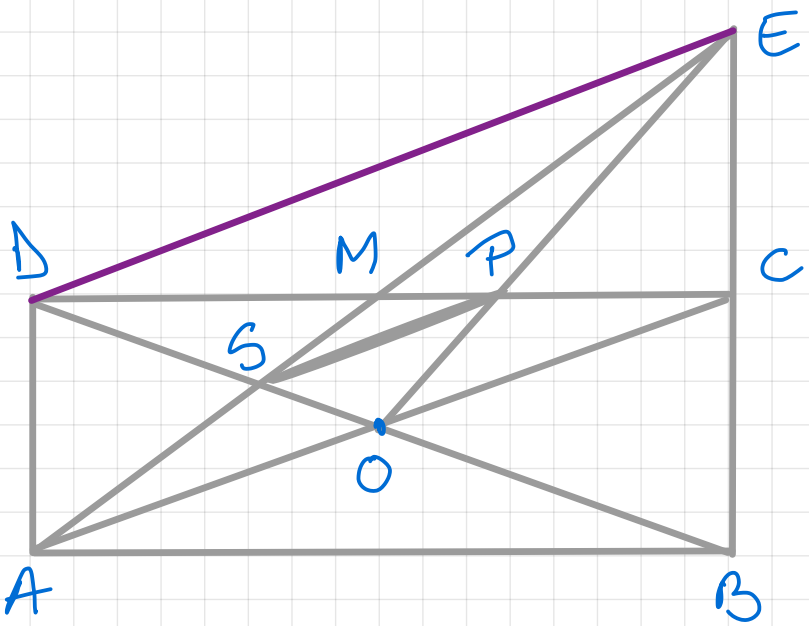
$BC^2 = 90$

$BC = \sqrt{90}$

$BC = 3\sqrt{10}$ cm.

$A_{ABCD} = AB \cdot BC$

$= 9\sqrt{10} \cdot 3\sqrt{10} = 270$ cm^2.



b)

$\hat{M}$ în ΔACE:

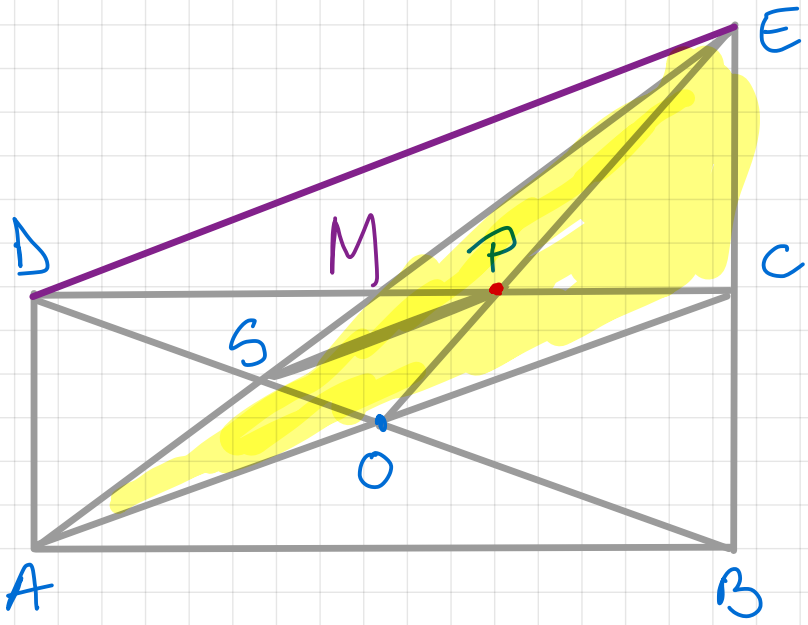
EO - mediană

CM - mediană

$EO \cap CM = \{P\}$ $\Rightarrow$

P - centrul de greutate al $\Delta ACE \Rightarrow$

$\Rightarrow \frac{MP}{MC} = \frac{1}{3}$ (1)

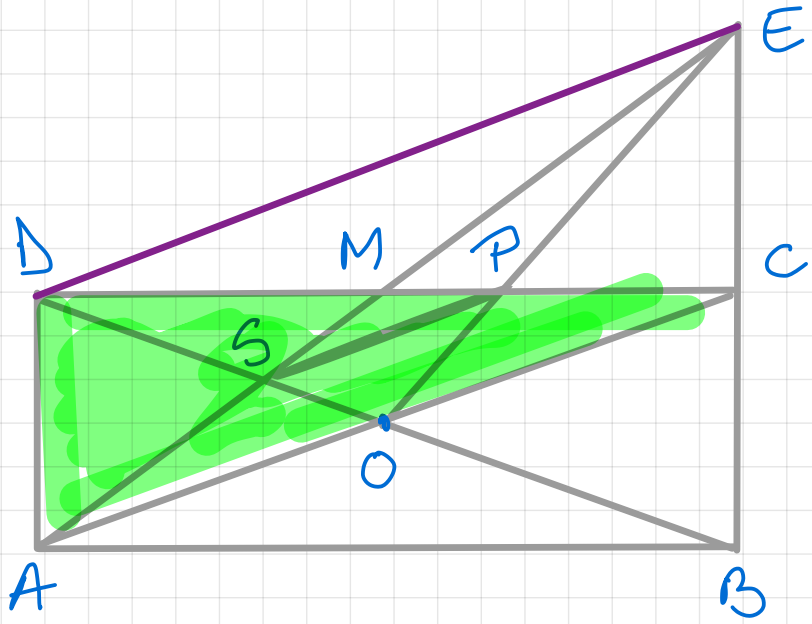


$\hat{I}n\ \triangle ABC$:

AM - mediană
BO - mediană
$AM \cap BO = \{S\}$ $\Rightarrow$

S - centrul de greutate al $\triangle ABC \Rightarrow$

$\Rightarrow \frac{SM}{AM} = \frac{1}{3}$ (2)



Din (1) și (2) $\Rightarrow \frac{MP}{MC} = \frac{SM}{AM} \Rightarrow SP \parallel AC$ (reciproca teoremei lui Thales)

$SP \parallel AC \Rightarrow \triangle MSP \sim \triangle MAC$ (teorema fundamentală a asemănării) $\Rightarrow \frac{SP}{AC} = \frac{MP}{MC} = \frac{SM}{AM} = i$

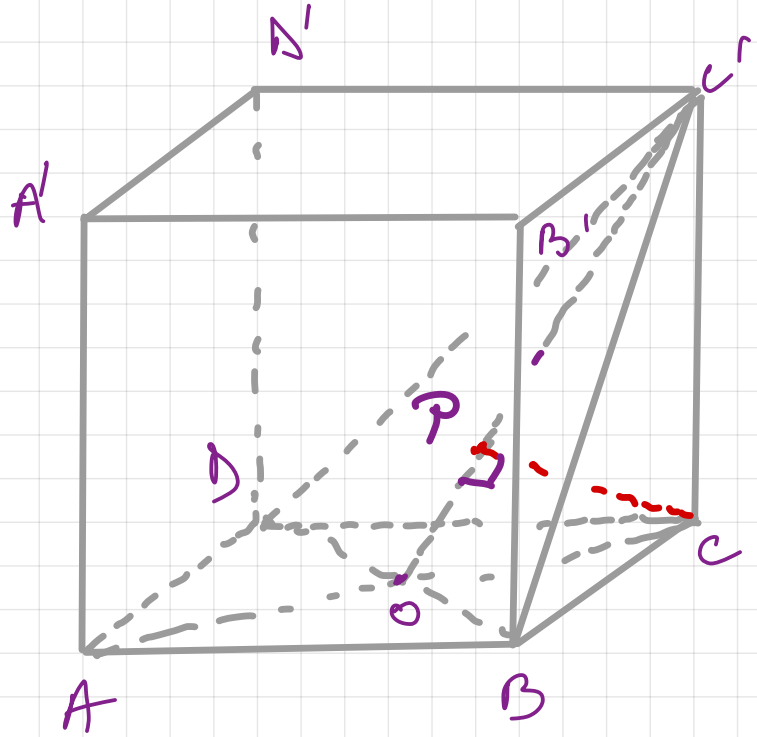
$\Rightarrow \frac{SP}{30} = \frac{1}{3} \Rightarrow SP = \frac{30 \cdot 1}{3} \Rightarrow SP = 10\ cm$

⑥ a) $BC' \parallel AD' \Rightarrow \sphericalangle(AB'; BC') = \sphericalangle(AB'; AD') =$

$= \sphericalangle B'AD'$

$\triangle B'AD'$ - echilateral $\Rightarrow \sphericalangle B'AD' = 60°$

b)



Fie $AC \cap BD = \{O\}$

Ducem $CP \perp C'O$

$CP \perp C'O$
$C'O \subset (BC'D)$
$C'O \perp DB$
$DB \subset (BC'D)$
$CO \perp DB$

$\Rightarrow CP \perp (BDC') \Rightarrow d(C;(BDC')) = CP$

$AB = 10$ cm $\Rightarrow AC = 10\sqrt{2}$ cm $\Rightarrow CO = 5\sqrt{2}$ cm

$\hat{I}n \triangle C'CO$:

$C'O^2 = C'C^2 + CO^2$

$C'O^2 = 10^2 + (5\sqrt{2})^2$

$C'O = \sqrt{150} \Rightarrow C'O = 5\sqrt{6}$ cm

$CP = \frac{CC' \cdot CO'}{C'O} \Rightarrow CP = \frac{10 \cdot 5\sqrt{2}}{5\sqrt{6}} \Rightarrow CP = \frac{10\sqrt{3}}{3}$ cm