

Concursul județean de matematică
”Sorin Simion”
 clasa a VII-a
 6 aprilie 2019
Barem

Subiectul 1

a)

$$\left. \begin{array}{l} 11 \cdot \overline{a0b} = 121 \cdot c^2 \\ \overline{a0b} = 11 \cdot c^2 \end{array} \right\} \quad 2p$$

Finalizare $\overline{abc} = 748$ 2p

b)

$$4n^2 - 12n + 64 = k^2, k \in \mathbb{Z} \quad 1p$$

$$(2n - 3 - k)(2n - 3 + k) = -55 \quad 1p$$

Finalizare 1p

Subiectul 2

Din inegalitatea mediilor avem

$$\sqrt{(a+1)(a+2)} \leq \frac{2a+3}{2} \quad 1p$$

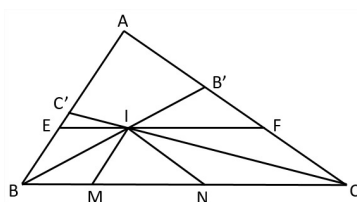
$$\frac{\sqrt{(a+1)(a+2)}}{2a+3} \leq \frac{1}{2} \quad 1p$$

Analog $\sqrt{(a+2)(a+3)} \leq \frac{2a+5}{2} \quad 1p$

$$\frac{\sqrt{(a+2)(a+3)}}{2a+5} \leq \frac{1}{2} \quad 1p$$

și $\frac{\sqrt{(a+3)(a+4)}}{2a+7} \leq \frac{1}{2} \quad 1p$

Finalizare 2p



Subiectul 3

a)

Fie $IM \parallel AB$ și $IN \parallel AC$, $M, N \in (BC)$

$\frac{BI}{BB'} = \frac{BN}{BC}$ și $\frac{CI}{CC'} = \frac{CM}{BC}$	1p
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$\frac{CI}{CC'} + \frac{BI}{BB'} = \frac{BN+CM}{BC} = \frac{BC+MN}{BC} > 1$	1p
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$$\left. \begin{array}{l} \angle EBI \equiv \angle IBM \\ \angle EBI \equiv \angle BIM(\text{alt int}) \end{array} \right\} \Rightarrow$$

$\Rightarrow \angle BIM \equiv \angle IBM \Rightarrow \triangle BIM$ isoscel $\Rightarrow (IM) \equiv (BM)$	1p
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Analog $(IN) \equiv (NC)$	1p
$\frac{BI}{BB'} + \frac{CI}{CC'} < \frac{3}{2} \iff \frac{BN+CM}{BC} < \frac{3}{2} \iff 2(BC + MN) < 3BC \iff$	1p
$2MN < BC \iff MN < BM + CN \iff MN < IM + IN(A)$	1p

b)

Presupunem că $IG \parallel BC$

Construim $EF \parallel BC$, $I \in EF$, $E \in (AB)$, $F \in (AC)$,
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Cum $IG \parallel BC \Rightarrow G \in EF$
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Fie mediana AA' , $\frac{AG}{AA'} = \frac{2}{3}$, $\frac{AE}{AB} = \frac{2}{3}$, $\frac{EF}{BC} = \frac{2}{3}$	1p
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$BMIE$ și $NCFI$ paralelograme $\Rightarrow BM = EI$, $IF = NC$
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$2BC = 3EF \Rightarrow EF = 2MN$

$\Rightarrow \frac{BI}{BB'} + \frac{CI}{CC'} = \frac{4MN}{3MN} = \frac{4}{3}$	1p
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Presupunem că $\frac{BI}{BB'} + \frac{CI}{CC'} = \frac{4}{3}$

$\Rightarrow EF = 2MN$

$\Rightarrow \frac{EF}{BC} = \frac{2}{3} \Rightarrow G$ se află pe $EF \Rightarrow GI \parallel BC$	1p
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