FORMULE TRIGONOMETRICE

	0	C_I	$\frac{\pi}{2}$	C_{II}	π	C_{III}	$\frac{3\pi}{2}$	C_{IV}	2π
sin x	0	+	1	+	0	-	-1	-	0
cos x	1	+	0	-	-1	-	0	+	1
tg x	0	+	$\infty \mid -\infty$	-	0	+	$\infty \mid -\infty$	-	0
ctg x	$\mid\infty$	+	0	-	$-\infty \mid \infty$	+	0	-	$-\infty \mid$

tgx=$\frac{\sin x}{\cos x}$

ctgx=$\frac{\cos x}{\sin x}$

tgx=$\frac{1}{ctx}$

ctgx=$\frac{1}{tgx}$

secx=$\frac{1}{\cos x}$

cosecx=$\frac{1}{\sin x}$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tg	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
ctg	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

$\sin\left(\frac{\pi}{2}-x\right)$=cosx

$\cos\left(\frac{\pi}{2}-x\right)$=sinx

$tg\left(\frac{\pi}{2}-x\right)$=ctgx

$ctgx\left(\frac{\pi}{2}-x\right)$=tgx

Formula fundamentala:

$\sin^2 x+\cos^2 x=1$

Formule provenite din formula fundamentala:

$\cos^2 x=1-\sin^2 x$

$tg^2 x=\frac{\sin^2 x}{1-\sin^2 x}$

$ctg^2 x=\frac{1-\sin^2 x}{\sin^2 x}$

$\sin^2 x=1-\cos^2 x$

$tg^2 x=\frac{1-\cos^2 x}{\cos^2 x}$

$ctg^2 x=\frac{\cos^2 x}{1-\cos^2 x}$

$\sin^2 x=\frac{tg^2 x}{1+tg^2 x}$

$\cos^2 x=\frac{1}{1+tg^2 x}$

$ctg^2 x=\frac{1}{tg^2 x}$

$\sin^2 x=\frac{1}{1+ctg^2 x}$

$\cos^2 x=\frac{ctg^2 x}{1+ctg^2 x}$

$tg^2 x=\frac{1}{ctg^2 x}$

Funcții trigonometrice:

f: $\mathbb{R} \to [-1,1]$, f(x) = sinx

f: $\mathbb{R} \to [-1,1]$, f(x) = cosx

f: $\mathbb{R} \setminus \left\{\frac{\pi}{2}+k\pi / k \in Z\right\} \to \mathbb{R}$, f(x) =tgx

f: $\mathbb{R} \setminus \{k\pi / k \in Z\} \to \mathbb{R}$, f(x)= ctgx

f:[-1,1] $\to \mathbb{R}$, f(x)= arcsin x

f:[-1,1] $\to \mathbb{R}$, f(x)= arccos x

f: $\left(-\frac{\pi}{2};\frac{\pi}{2}\right) \to \mathbb{R}$, f(x)= arctg x

f: $(0;\pi) \to \mathbb{R}$, f(x)= arcctg x

Paritatea si imparitatea functiilor trigonometrice:

sin(-x) = - sinx
cos(-x) = cosx
tg(-x) = - tgx
ctg(-x) = - ctgx

arcsin(-x)= -arcsin x
arccos(-x)= π -arccos x
arctg(-x)= -arctg x
arcctg(-x)= π -arcctg x

$x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right] \Rightarrow$ arcsin(sinx)=x

$x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right] \Rightarrow$ arccos(cosx)=x

$x \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right) \Rightarrow$ arctg(tgx)=x

$x \in (0;\pi) \Rightarrow$ arcctg(ctgx)=x

$x \in [-1, 1] \Rightarrow$ sin(arcsinx)=x
$x \in [-1, 1] \Rightarrow$ cos(arccosx)=x
$x \in \mathbb{R} \Rightarrow$ tg(arctgx)=x
$x \in \mathbb{R} \Rightarrow$ ctg(arcctgx)=x

Periodicitatea functiilor trigonometrice:

sin(x+2kπ) = sinx
cos(x+2kπ) = cosx
tg(x+kπ) = tgx
ctg(x+kπ) = ctgx,
$k \in \mathbf{Z}$

Reducerea la primul cadran:

$x \in C_{II}$:
- sinx = sin(π - x)
- cosx = - cos(π - x)
- tgx = - tg(π - x)
- ctgx = - ctg(π - x)

$x \in C_{III}$:
- sinx = - sin(x - π)
- cosx = - cos(x - π)
- tgx = tg(x - π)

$x \in C_{IV}$:
- sinx = - sin(2π - x)
- cosx = cos(2π - x)
- tgx = - tg(2π - x)
- ctgx = - ctg (2π - x)

Deplasarea in punctul diametral opus:

x ∈ □ :
- sin(x - π) = sin(x+π) = - sinx
- cos(x - π) = cos(x+π) = - cosx
- tg(x - π) = tg(x+π) = tgx
- ctg(x - π) = ctg(x+π) = ctgx

sin(x+y) = sinxcosy + cosxsiny
cos(x+y) = cosxcosy – sinxsiny

$$tg(x+y) = \frac{tgx + tgy}{1 - tgxtgy}$$

$$ctg(x+y) = \frac{ctgx \cdot ctgy - 1}{ctx + ctgy}$$

sin(x-y) = sinxcosy – cosxsiny
cos(x-y) = cosxcosy + sinxsiny

$$tg(x-y) = \frac{tgx - tgy}{1 + tgxtgy}$$

$$ctg(x-y) = \frac{-ctgxctgy - 1}{ctgx - ctgy}$$

sin2x = 2sinxcosx

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$tg2x = \frac{2tgx}{1 - tg^2 x}$$

$$ctg2x = \frac{ctg^2 x - 1}{2ctgx}$$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

$$tg^2 \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$$

$$ctg^2 \frac{x}{2} = \frac{1 + \cos x}{1 - \cos x}$$

$$\cos x - 1 = -2\sin^2 \frac{x}{2}$$

$$\cos x + 1 = 2\cos^2 \frac{x}{2}$$

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = -3\cos x + 4\cos^3 x$$

$$tg3x = \frac{3tgx - tg^3 x}{1 - 3tg^2 x}$$

$$ctg3x = \frac{ctg^3 x - 3ctgx}{3ctg^2 x - 1}$$

Transformarea produselor in sume:

$$\cos x \cos y = \frac{\cos(x+y) + \cos(x-y)}{2}$$

$$\sin x \cos y = \frac{\sin(x+y) + \sin(x-y)}{2}$$

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$arctg\, x \pm arctg\, y = arctg \frac{x \pm y}{1 \mp xy}$$

Transformarea sumelor in produse:

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$tgx + tgy = \frac{\sin(x+y)}{\cos x \cos y}; \quad tgx - tgy = \frac{\sin(x-y)}{\cos x \cos y}$$

Substitutia universala:

$$t = tg\frac{x}{2} \Rightarrow$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$tgx = \frac{2t}{1-t^2}$$

$$ctgx = \frac{1-t^2}{2t}$$

Ecuatii trigonometrice:

- sinx = a, $a \in [-1, 1] \Rightarrow x = (-1)^k \arcsin a + k\pi$, $k \in \mathbf{Z}$
- cosx = a, $a \in [-1, 1] \Rightarrow x = \pm \arccos a + 2k\pi$, $k \in \mathbf{Z}$
- tgx = a, $a \in \mathbf{R} \Rightarrow x = arctg\, a + k\pi$, $k \in \mathbf{Z}$
- ctgx = a, $a \in \mathbf{R} \Rightarrow x = arcctg\, a + k\pi$, $k \in \mathbf{Z}$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$arctg\, x + arcctg\, x = \frac{\pi}{2}$$

- sinx = sina, $a \in \mathbf{R} \Rightarrow x = (-1)^k a + k\pi$, $k \in \mathbf{Z}$
- cosx = cosa, $a \in \mathbf{R} \Rightarrow x = \pm a + 2k\pi$, $k \in \mathbf{Z}$
- tgx = tga, $a \in \mathbf{R} \setminus \left\{\frac{\pi}{2} + k\pi / k \in Z\right\} \Rightarrow x = a + k\pi$, $k \in \mathbf{Z}$
- ctgx = ctgx, $a \in \mathbf{R} \setminus \{k\pi / k \in Z\} \Rightarrow x = a + k\pi$, $k \in \mathbf{Z}$

- sinx = 0 $\Rightarrow x = k\pi$, $k \in \mathbf{Z}$
- cosx = 0 $\Rightarrow x = \frac{\pi}{2} + k\pi$, $k \in \mathbf{Z}$
- tgx = 0 $\Rightarrow x = k\pi$, $k \in \mathbf{Z}$
- ctgx = 0 $\Rightarrow x = \frac{\pi}{2} + k\pi$, $k \in \mathbf{Z}$