

GEOMETRIE ANALITICA IN PLAN

(clasa a 11-a)

$A(x_A, y_A)$

$x_A > 0, y_A > 0 \Rightarrow A \in C_I$

$x_A < 0, y_A > 0 \Rightarrow A \in C_{II}$

$x_A < 0, y_A < 0 \Rightarrow A \in C_{III}$

$x_A > 0, y_A < 0 \Rightarrow A \in C_{IV}$

$x_A > 0, y_A = 0 \Rightarrow A \in (Ox)$

$x_A < 0, y_A = 0 \Rightarrow A \in (Ox')$

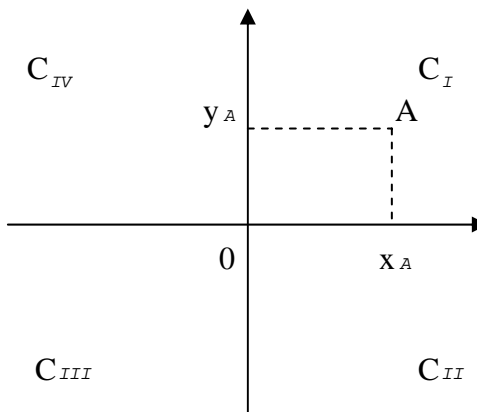
$x_A = 0, y_A > 0 \Rightarrow A \in (Oy)$

$x_A = 0, y_A < 0 \Rightarrow A \in (Oy')$

$x_A = y_A = 0 \Rightarrow A = O$

$x_A \in \mathbf{R}, y_A = 0 \Rightarrow A \in Ox$

$x_A = 0, y_A \in \mathbf{R} \Rightarrow A \in Oy$



Distanța dintre două puncte $A(x_A, y_A), B(x_B, y_B)$

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

Mijlocul unui segment $M \in [AB]$ astfel încât $[AM] \equiv [MB]$

$$x_M = \frac{x_A + x_B}{2}, y_M = \frac{y_A + y_B}{2}$$

Punctul care împarte un segment într-un raport dat $D \in [AB]$ astfel încât $\frac{AD}{DB} = k$

$$x_D = \frac{x_A + kx_B}{1+k}, y_D = \frac{y_A + ky_B}{1+k}$$

Centrul de greutate G al unui triunghi ABC :

$$x_G = \frac{x_A + x_B + x_C}{3}, y_G = \frac{y_A + y_B + y_C}{3}$$

Panta unei drepte

$m_d = \operatorname{tg} \alpha$, unde α = unghiul dintre axa $[Ox]$ și d , considerat în sens trigonometric, dacă $d \parallel Oy$

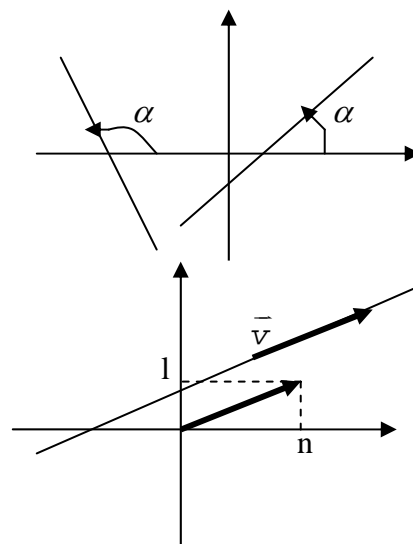
Observație: $m_d = 0 \Leftrightarrow d \parallel Ox$

Panta dreptei determinată de punctele A și B : $m_{AB} = \frac{y_A - y_B}{x_A - x_B}$

Panta dreptei determinată de direcția vectorului $\vec{v}(l, n)$

$$\vec{v} = l\vec{i} + n\vec{j} \text{ este } m_d = \frac{n}{l}$$

Panta dreptei $ax + by + c = 0$ este $-\frac{a}{b}$.



Ecuatia dreptei

Ecuatia dreptei de panta m si care trece prin punctul $A(x_A, y_A)$

$$d: y - y_A = m(x - x_A)$$

Ecuatia explicita

d: $y = mx + n$, unde $m =$ **panta dreptei**, $n =$ **ordonata la origine**

Ecuatia dreptei care trece prin doua puncte $A(x_A, y_A)$, $B(x_B, y_B)$

$$AB: \frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A}, x_A \neq x_B, y_A \neq y_B$$

daca $x_A = x_B \Rightarrow AB // Oy$ AB are ecuatia $x = x_A$

daca $y_A = y_B \Rightarrow AB // Ox$ AB are ecuatia $y = y_A$

Ecuatia dreptei de directie $\vec{v}(l, n)$ si care trece prin $A(x_A, y_A)$

$$AB: \frac{x - x_A}{l} = \frac{y - y_A}{n}$$

Ecuatiile parametrice ale unei drepte

$$AB: \frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A} = t \Rightarrow AB: \begin{cases} x = (x_B - x_A)t + x_A \\ y = (y_B - y_A)t + y_A \end{cases}$$

$$AB: \frac{x - x_A}{l} = \frac{y - y_A}{n} = t \Rightarrow AB: \begin{cases} x = lt + x_A \\ y = nt + y_A \end{cases}$$

Ecuatia sub forma de determinant

$$AB: \begin{vmatrix} x & y & 1 \\ x_A & y_A & 1 \\ x_B & y_B & 1 \end{vmatrix} = 0$$

Ecuatia prin taieturi

$$A(a, 0), B(0, b) \Rightarrow AB: \frac{x}{a} + \frac{y}{b} = 1$$

Ecuatia generala a dreptei

d: $ax + by + c = 0$, $a, b, c \in \mathbf{R}$, a si b nu sunt simultan nule

Observatii: $a = 0 \Rightarrow b \neq 0 \Rightarrow d: y = -\frac{c}{b} \Rightarrow d // Ox$

$b = 0 \Rightarrow a \neq 0 \Rightarrow d: x = -\frac{c}{a} \Rightarrow d // Oy$

$a \neq 0, b \neq 0 \Rightarrow d // Ox, d // Oy$

$c = 0 \Rightarrow O \in d$

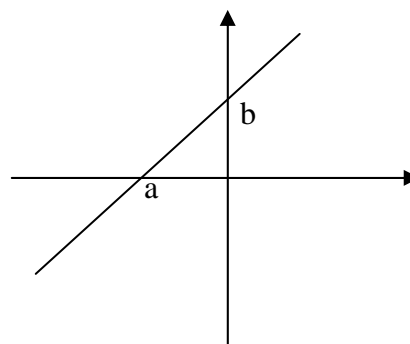
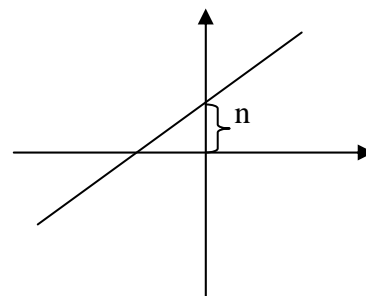
Panta dreptei este $m_d = -\frac{a}{b}$ (coeficientul lui x cand izolam pe y)

Directia dreptei este $\vec{v} = (ak, bk)$, $k \in \mathbf{R}^*$




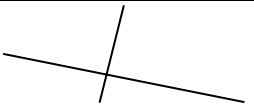
Conditia de apartenenta la o dreapta

$A(x_A, y_A)$, d: $ax + by + c = 0$

$A \in d \Rightarrow ax_A + by_A + c = 0$



Pozitii relative ale dreptelor

	$d_1: a_1x + b_1y + c_1 = 0$ $d_2: a_2x + b_2y + c_2 = 0$	$d_1: y = m_1x + n_1$ $d_2: y = m_2x + n_2$
$d_1 = d_2$ 	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	$\begin{cases} m_1 = m_2 \\ n_1 = n_2 \end{cases}$
$d_1 // d_2$ 	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	$\begin{cases} m_1 = m_2 \\ n_1 \neq n_2 \end{cases}$
$d_1 \neq d_2$ $d_1 \cap d_2 \neq \emptyset$ 	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	$m_1 \neq m_2$
$d_1 \perp d_2$ 	$a_1a_2 + b_1b_2 = 0$	$m_1 m_2 = -1$

Conditia de coliniaritate

A, B, C sint coliniare $\Leftrightarrow A \in BC$

$$\text{sau } \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$$

Conditia de concurenta a trei drepte

$$d_1: a_1x + b_1y + c_1 = 0$$

$$d_2: a_2x + b_2y + c_2 = 0$$

$$d_3: a_3x + b_3y + c_3 = 0$$

$$d_1, d_2, d_3 \text{ sunt concurente} \Leftrightarrow \begin{cases} \{A\} = d_1 \cap d_2 \\ A \in d_3 \end{cases}$$

$$\text{sau } \text{rang} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ c_1 & c_2 \end{pmatrix} = 0 \text{ si } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Unghiul dintre doua drepte de pante m_1, m_2

$$\text{tg } \varphi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|, \varphi \in \left[0, \frac{\pi}{2} \right)$$

Distanta de la un punct A la o dreapta h

$A(x_A, y_A)$, $h: ax + by + c = 0$

$$d(A, h) = \frac{|ax_A + by_A + c|}{\sqrt{a^2 + b^2}}$$

Aria unui triunghi ABC

$$\Delta = \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} \quad A_{\Delta ABC} = \frac{1}{2} |\Delta|$$

