

Mathematica Centrum

Together, let's shape the mathematicians of the future

A. You will need:

1. A **blank response** form with the title "Mathematica", identical (except for the title) to the ones which are used for the Contests. Download this form and make as many copies as you need, so that your students can write the preparatory test and learn how to properly fill out a response form. (Remember that these copies cannot be used for the Contests. Your students will use the response forms that will be sent to you. Each student registered for a contest will receive a response form corresponding to the contest that he is writing. The only reason you are doing these copies is to show your students how to properly fill out a response form).
2. The **preparatory test** (this document), which your students can write to become familiar with multiple choice questions. Download this test and make as many copies as you need. (Remember that you are doing this to explain to your students the purpose of the preparatory test. The preparatory test defines the type of problems that appear in the actual contest.)
3. The **answer key**. Download the answer key and make as many copies as you need.

B. How to fill out a response form properly:

Use an **HB pencil** for coding all parts of the form. Do not use a ball point pen or felt-tip marker.

In the box at the top part of the form, tell your students to **PRINT** their school's name in full as well as their city/town and province. To the right of the box, tell them to **PRINT** their date of birth and sign their name to certify that the answers given represent their own work.

In the box on the mid-left of the form, tell your students to **PRINT** their last and first names. Tell them to code each letter by filling in the appropriate circle under each letter. (If your last name is Mathews, first you code the letter M by filling in the circle containing the M right under the letter M of Mathews, then you code the A by filling in the circle containing the A right under the letter A of Mathews. Do this for every other letter of your last name and for each letter of your first name). If the last name of a student is hyphenated, for example Jones-Smith, or if his/her first name is hyphenated, like Carol-Ann, inform the student to simply write Jones Smith and Carol Ann.

The mid-right part of the form outlines important instructions which are a reminder of what to do to code the response form correctly. The lower part of this box shows examples of incorrect coding. Remind your students to **completely** fill in each circle.

The box at the bottom of the form is made of circles which your students will fill in to record their answers to the questions. Again, tell them to fill each circle completely!

C. Problems:

Allow your students to write the preparatory test to be sure that they understand how to properly fill out the response form and to prepare them as to the type of problems that appear in the actual contests. It is important that your students do the problems intended for them :

Newton : all of the problems (except #21 and #30)

Lagrange : problems #1 to #25 (except #20)

Euler : problems #1 to #19 plus #24, #26, and #30

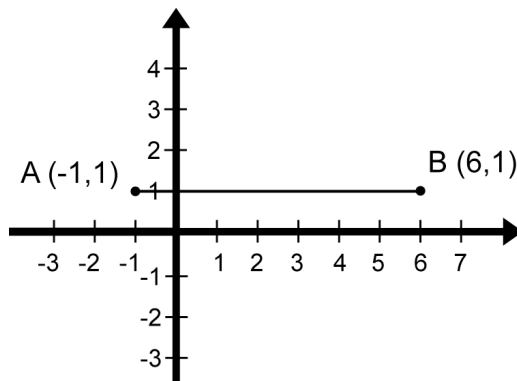
Mathematica Centrum

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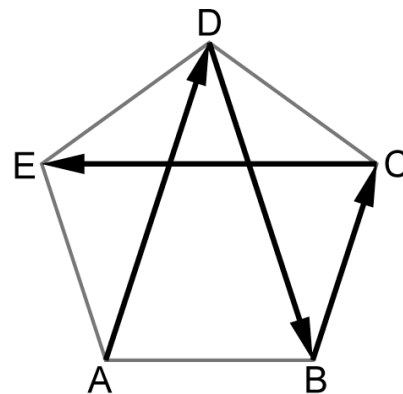
PREPARATORY TEST 2010

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

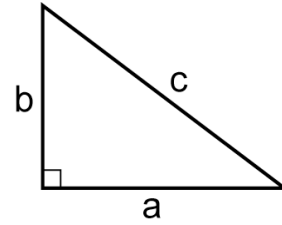
1. The value of $-5 + (-7) - (3 - 5)$ is
 A) -10 B) 10 C) -14 D) 8 E) -12
2. What is the value of $2 + 2^3 + \sqrt{16}$?
 A) 10 B) 12 C) 14 D) 16 E) 18
3. $(5/8 + 1/2) \div (7/16 - 2/8) = ?$
 A) $1/2$ B) 8 C) 4 D) 6 E) $27/108$
4. The largest prime factor of 310 is
 A) 2 B) 31 C) 5 D) 7 E) 11
5. If $x\%$ of 25 is 30, then $x\%$ of 20 is
 A) x B) 12 C) 0.2 D) 18 E) 24
6. The product of two natural numbers is 12. Their largest possible sum is
 A) 10 B) 11 C) 12 D) 14 E) 13
7. 1 L of liquid A has 10% more calories than 1 L of liquid B. How many calories does 1 L of liquid B contain, if 1 L of liquid A contains 660 calories?
 A) 600 calories B) 550 calories
 C) 726 calories D) 400 calories
 E) 500 calories
8. Line segment AB is reflected in the x-axis. The coordinates of the images of points A and B after the reflection are, respectively,
 A) $(-1,-1)$ et $(6,1)$ B) $(-1,-1)$ et $(6,-1)$
 C) $(-1,1)$ et $(1,6)$ D) $(1,-1)$ et $(-6,1)$
 E) $(-1,1)$ et $(6,-1)$



9. For how many values of the set $\{-4, -2, -1/2, 0, 1/4, 2/3\}$ is $x > x^2$?
- A) 6 B) 3 C) 2 D) 4 E) 5
10. The ages of 5 people vary from 49 to 90 years old. Which of the following choices could represent their average age?
- A) 84 B) 54 C) 57 D) 79 E) 82
11. Mathew has written all the natural numbers from 1 to 100. Which digit has he used exactly 11 times?
- A) 5 B) 6 C) 4 D) 1 E) 0
12. The average of $5/6$ and $8/12$ plus the average of $1/3$ and $1/2$ is equal to
- A) $4/3$ B) $7/6$ C) $7/12$ D) 1 E) $3/2$
13. What is the smallest positive integer that is divisible by 3, 4, 6, and 8?
- A) 30 B) 48 C) 24 D) 12 E) 36
14. A ball loses two thirds of its energy every time it hits a surface. After each bounce, it rises to one third the height from which it fell. After how many bounces will it rise to a height of less than 2 metres, if the ball initially falls from a height of 144 metres?
- A) 5 B) 4 C) 3 D) 6 E) 2
15. Five towns forming the vertices of a pentagon are joined by a highway system as shown in the diagram. The highway system is made of diagonal highways (like AD) and lateral highways (like BC). Starting from A, a salesman wants to go to all other towns (B, C, and D) only once before he goes to E. The salesman must always travel in a straight line when going from one town to another. The diagram shows one of the paths (in bold) that the salesman can follow to go from A to E (ADBCE). In total, how many different ways can the salesman drive from A to E?
- A) 4 B) 5 C) 6 D) 7 E) 10
16. The result of $(4 - 3) + (5 - 4) + (6 - 5) + \dots + (103 - 102)$ is
- A) 100 B) 101 C) 99 D) 98 E) 102
17. How many 0's are used when $10^{20} + 999$ is written as a natural number?
- A) 17 B) 18 C) 16 D) 19 E) 20



18. Pythagoras' theorem states that, in a right triangle, if c is the length of the hypotenuse and a and b are the lengths of the other two sides, then $a^2 + b^2 = c^2$. If $a = 6$ and $b = 8$, then $c^2 = 6^2 + 8^2 = 100$ and $c = \sqrt{100} = 10$. What is the length of c if $b = 1$ and $a = 1$?

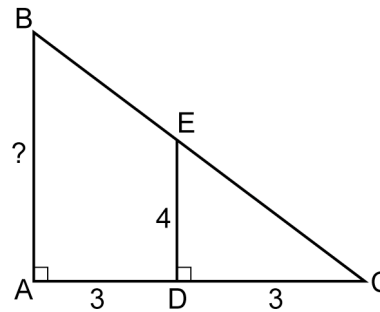


- A) 2 B) 1.5 C) $\sqrt{3}$
D) $\sqrt{2}$ E) 0.41

19. The height of a rectangle is 3 cm and its base is 5 cm. What is the area of the new rectangle when the height is doubled and the base is tripled?

- A) 15 cm^2 B) 65 cm^2 C) 60 cm^2
D) 90 cm^2 E) 42 cm

20. Two right triangles ABC and DEC are shown in the diagram. If $AD = 3$, $DC = 3$, and $DE = 4$, what is the length of line segment AB?



- A) 6 B) 10
C) 5 D) 6.5
E) 8

21. A bag contains x red balls and y green balls. If a ball is chosen at random, what is the probability that the ball is green?

- A) $x : (x + y)$ B) $y : (x + y)$ C) $xy : (x + y)$ D) $(x + y) : y$ E) $(x + y) : x$

22. If $3x + 2 = 2x - 2$, what is the value of $2x + 5$?

- A) 3 B) 1 C) -2 D) -1 E) -3

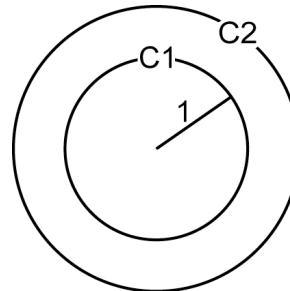
23. Two positive integers are in the ratio 5 : 3. If their difference is 12, what is their sum?

- A) 18 B) 30 C) 48 D) 40 E) 50

24. When the digits 1, 2, 3, and 4 are all used to form 4-digit natural numbers, 24 different numbers can be formed. If all these numbers are written in decreasing order (from the largest to the smallest), what number will be the 6th number on the list?

- A) 3 241 B) 3 421 C) 4 132 D) 4 123 E) 4 312

25. Two circles (C_1 and C_2) are concentric. The area of the large circle is triple the area of the smaller one. What is the difference between the radii of the two circles, if the radius of the smaller circle is 1?



- A) 2 B) $\sqrt{2}$ C) $\sqrt{3}$
D) $\sqrt{3} - 1$ E) $\sqrt{2} - 1$

26. Let m be a natural number. If we know that m^2 has 10 prime factors, how many prime factors does the number m have?

- A) 5 B) 4 C) 10 D) 6 E) 20

27. There are two numbers on the number line that are twice as far from 2 as from -1. If the number 0 is one of them, what is the other?

- A) $1/2$ B) $-3/4$ C) -4 D) -3 E) 6

28. What is the product of $(x + 2)(x + 2)$?

- A) $x^2 + 4x + 4$ B) $x^2 + 2$ C) $x^2 + x + 4$ D) $x^2 + 2x + 4$ E) $x^2 + 4$

29. The base of an isosceles triangle is 6 cm and its area is 12 cm^2 . What is its perimeter?

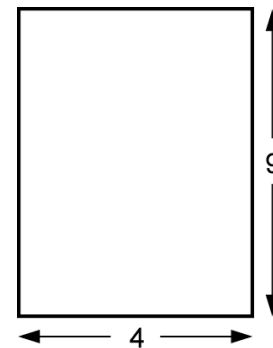
- A) 18 cm B) 15 cm C) 13 cm D) 12 cm E) 16 cm

30. Did you know that we can find the number of divisors of any natural number without knowing each one of its divisors? How? First, the number must be factored. Every natural number n can be factored as $n = p_1^{E_1} \times p_2^{E_2} \times p_3^{E_3} \dots \times p_n^{E_n}$. In this factorisation, the symbols $p_1, p_2, p_3, \dots, p_n$ represent the different prime factors of the factorisation, while $E_1, E_2, E_3, \dots, E_n$ represent the respective exponents of these prime factors. The factorisation of 24 is $2^3 \times 3^1$, that of 36 is $2^2 \times 3^2$. The exponents of the prime factors of a number enable us to find the number of its divisors by simply replacing the values of $E_1, E_2, \dots, E_n$ corresponding to that number in the expression: $(E_1 + 1) \times (E_2 + 1) \times (E_3 + 1) \dots \times (E_n + 1)$. The number of divisors of 24 is given by the product $(3 + 1) \times (1 + 1)$, which is 8. By the way, the 8 divisors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The number of divisors of 36 is given by the product $(2 + 1) \times (2 + 1)$, which is 9. The 9 divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. We can find the number of divisors of any natural number without knowing each one of its divisors. What is the number of divisors of 300?

- A) 9 B) 18 C) 12
D) 14 E) 4

31. The volume of a right cylinder is given by the formula $V = \pi r^2 h$ and that of a sphere by the formula $V = \frac{4}{3} \pi r^3$. What is the radius of the sphere that has the same volume as the right cylinder whose front view is shown in the diagram?

- A) π B) 2 C) 3
D) 3π E) $\sqrt{\pi}$



32. If $x/3 = y/4$, what is the value of $(8x + 6y) : 2y$?

- A) 4 B) 6 C) 5 D) 3 E) 7

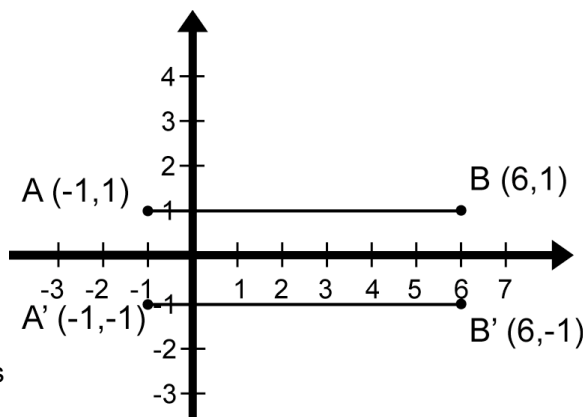
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PREPARATORY TEST 2010 COMPLETE SOLUTIONS

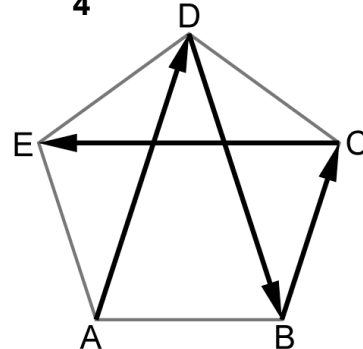
EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

1. The value of $-5 + (-7) - (3 - 5)$ is $-5 - 7 - (-2) = -12 + 2 = -10$.
2. The value of $2 + 2^3 + \sqrt{16}$ is $2 + 8 + 4 = 14$.
3. $(5/8 + 1/2) \div (7/16 - 2/8) = (5/8 + 4/8) \div (7/16 - 4/16) = 9/8 \div 3/16 = 9/8 \times 16/3 = 6$.
4. The prime factors of 310 are $2 \times 5 \times 31$. The largest prime factor of 310 is 31.
5. If $x\%$ of 25 is 30, we can write that $x\% \times 25 = 30$, $x\% = 30/25$, $x\% = 6/5$, $x\% = 120\%$, and $x = 120$. 120% of 20 = $1.2 \times 20 = 24$.
6. The product of two natural numbers is 12. These two numbers can be 1 and 12, 2 and 6, or 3 and 4. Their largest possible sum is $(1 + 12) = 13$.
7. If 1 L of liquid A contains 10% more calories than 1 L of liquid B, then 1 L of liquid A contains 1.1 times more calories than 1 L of liquid B. If 1 L of liquid A contains 660 calories, then 1 L of liquid B contains $(660 \div 1.1) = 600$ calories.
8. Line segment AB is reflected in the x-axis. The coordinates of the images of points A and B after the reflection are, respectively, $(-1, -1)$ and $(6, -1)$.
9. The only values for which $x > x^2$ are $1/4$ and $2/3$ ($1/4 > 1/16$ and $2/3 > 4/9$). The values for which $x < x^2$ are -4 , -2 , and $-1/2$ ($-4 < 16$, $-2 < 4$, and $-1/2 < 1/4$). For $x = 0$, $0 = 0^2$.
10. The minimal average age of these 5 people is $(4 \times 49 + 90) \div 5 = 57.2$ years. The maximum average is $(4 \times 90 + 49) \div 5 = 81.8$ years. The only choice that could represent their average age is 79 years.
11. The only digit that he used exactly 11 times is the 0 (**10**, **20**, **30**, ... **100**).
12. The average of $5/6$ and $8/12$ is $((10/12 + 8/12) \div 2) = 9/12$. The average of $1/3$ and $1/2$ is $((2/6 + 3/6) \div 2) = 5/12$. The sum of the two averages is $(9/12 + 5/12) = 14/12$ or $7/6$.

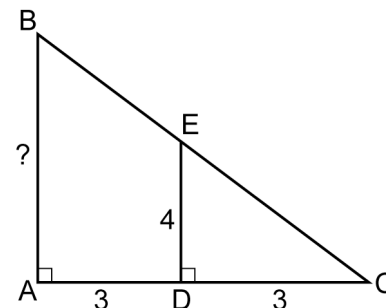
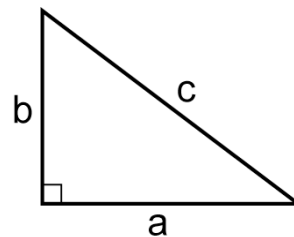


13. The smallest positive integer that is divisible by 3, 4, 6, and 8 is the LCM of these numbers. The LCM of 3 (3), 4 (2×2), 6 (2×3), and 8 ($2 \times 2 \times 2$) is ($2 \times 2 \times 2 \times 3$) 24.
14. Look at the diagram below. After each bounce, the ball rises to one third the height from which it fell. After the first bounce, it rises to a height of 48 metres. After the second, it rises to a height of 16 metres, It will rise to a height of less than 2 metres after 4 bounces.

144 48 16 5.33 1.78
1 2 3 4

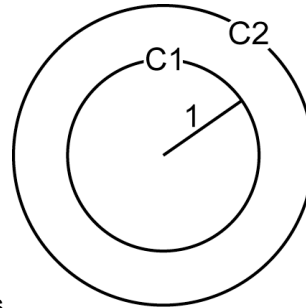


15. The diagram shows one of the paths (in bold) that the salesman can follow to go from A to E (ADBCE). In all, he can drive from A to E in 6 different ways (ADBCE, ADCBE, ABCDE, ABDCE, ACDBE et ACBDE). There are 6 different ways to go from A to E because there are 6 ($1 \times 2 \times 3$) different ways to arrange the letters BCD.
16. The result of $(4 - 3) + (5 - 4) + (6 - 5) + \dots + (103 - 102)$ is 100. We know that each parenthesis is worth 1. We must find out how many parentheses there are. First, we can write this series as $(4 - 3) + (5 - 4) + (6 - 5) + \dots + (100 - 99) + (101 - 100) + (102 - 101) + (103 - 102)$. The last 3 parentheses are equivalent to $(1 - 0) + (2 - 1) + (3 - 2)$. If these 3 parentheses are placed at the beginning of the series, we get the following series: $(1 - 0) + (2 - 1) + (3 - 2) + (4 - 3) + (5 - 4) + (6 - 5) + \dots + (100 - 99)$. This series is composed of exactly 100 terms. Each term being equal to 1, the result is 100.
17. When $10^{20} + 999$ is written as a natural number, we get the number (999 + 100 000 000 000 000 000 000) 100 000 000 000 000 000 999. This number has 17 zeros.
18. Pythagoras' theorem states that, in a right triangle, if c is the length of the hypotenuse and a and b represent the lengths of the other two sides, then $a^2 + b^2 = c^2$. If $b = 1$ and $a = 1$, then $c^2 = 1^2 + 1^2 = 2$ and $c = \sqrt{2}$.
19. The height of a rectangle is 3 cm and its base is 5 cm. When the height is doubled (6 cm) and the base is tripled (15 cm), the area of the new rectangle is (6 cm $\times$ 15 cm) 90 cm².
20. Triangles ABC and DEC are similar (point C is the centre of enlargement). The scale factor is 2, because $AC : DC = 2$. The length of line segment AB is ($2 \times ED = 2 \times 4$) 8.
21. A bag contains x red balls and y green balls. If a ball is chosen at random, the probability that the ball is green is obviously $y : (x + y)$.
22. From $3x + 2 = 2x - 2$, we get $x = -4$. The value of $2x + 5$ is $2(-4) + 5 = -3$.



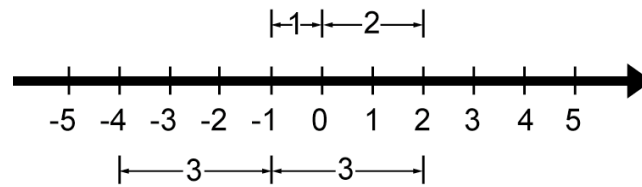
23. Two positive integers are in the ratio 5 : 3. If the smaller one is x , then the larger one is $x + 12$. We can write the equation $(x + 12) : x = 5 : 3$. This equation becomes $3(x + 12) = 5x$, from which we get $x = 18$. The sum of these integers is $(18 + 30) = 48$.
24. The largest of these numbers is 4 321. The 2nd is 4 312, the 3rd is 4 231, the 4th is 4 213, the 5th is 4 132, and the 6th is 4 123.

25. The area of the smaller circle is $\pi \cdot 1^2 = \pi$. The area of the large circle is $3 \times \pi$. The radius of the large circle is given by the equation: $\pi r^2 = 3\pi$. We find $r = \sqrt{3}$. The difference between the two radii is $\sqrt{3} - 1$. By the way, this value represents the smallest possible distance between a point on C_1 and a point on C_2 .



26. Let us suppose that m is 20. The number of prime factors of 20 is $(2 \times 2 \times 5) = 3$. The number of prime factors of 400 is $(2 \times 2 \times 2 \times 2 \times 5 \times 5) = 6$. The number of prime factors of 20^2 is double that of 20. The number of prime factors of m^2 is always double that of m . If the number m^2 has 10 prime factors, then m has 5.

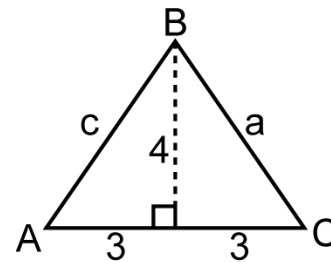
27. Look at the number line shown in the diagram. The two numbers that are twice as far from 2 as from -1 are 0 and -4.



28. The product of $(x + 2)(x + 2)$ is $x^2 + 4x + 4$.

$$(x + 2)(x + 2) = x^2 + 4x + 4$$

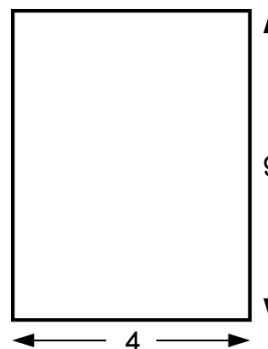
29. If the base b is 6 cm and the area is 12 cm^2 , the height h is $(6 \times h \div 2 = 12) = 4$ cm. If the values of a and c are equal, we can find the perimeter of the triangle by finding the value of a . We know that $a^2 = 3^2 + 4^2$. We get $a = 5$ cm. The perimeter of the triangle is $(6 + 5 + 5) = 16$ cm.



30. $300 = 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3 \times 5^2$. The number of divisors of 300 is given by the product $(2 + 1) \times (1 + 1) \times (2 + 1)$, which is 18.

31. The volume of the right cylinder is $\pi \times 2^2 \times 9$. The volume of the sphere is $\frac{4}{3} \pi r^3 = 36 \pi$. We find that $r^3 = 27$ and $r = 3$.

32. From $x/3 = y/4$, we get $4x = 3y$ and $8x = 6y$. The value of $(8x + 6y) : 2y$ is $(6y + 6y) : 2y = 6$.



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- | | | | | |
|------------------------|------------------------|------------------------|--------------------------|--------------------------|
| 1 ● (B) (C) (D) (E) | 11 (A) (B) (C) (D) ● | 21 (A) ● (C) (D) (E) | 31 (A) (B) ● (D) (E) | 41 (A) (B) (C) (D) (E) |
| 2 (A) (B) ● (D) (E) | 12 (A) ● (C) (D) (E) | 22 (A) (B) (C) (D) ● | 32 (A) ● (C) (D) (E) | 42 (A) (B) (C) (D) (E) |
| 3 (A) (B) (C) ● (E) | 13 (A) (B) ● (D) (E) | 23 (A) (B) ● (D) (E) | 33 (A) (B) (C) (D) (E) | 43 (A) (B) (C) (D) (E) |
| 4 (A) ● (C) (D) (E) | 14 (A) ● (C) (D) (E) | 24 (A) (B) (C) ● (E) | 34 (A) (B) (C) (D) (E) | 44 (A) (B) (C) (D) (E) |
| 5 (A) (B) (C) (D) ● | 15 (A) (B) ● (D) (E) | 25 (A) (B) (C) ● (E) | 35 (A) (B) (C) (D) (E) | 45 (A) (B) (C) (D) (E) |
| 6 (A) (B) (C) (D) ● | 16 ● (B) (C) (D) (E) | 26 ● (B) (C) (D) (E) | 36 (A) (B) (C) (D) (E) | 46 (A) (B) (C) (D) (E) |
| 7 ● (B) (C) (D) (E) | 17 ● (B) (C) (D) (E) | 27 (A) (B) ● (D) (E) | 37 (A) (B) (C) (D) (E) | 47 (A) (B) (C) (D) (E) |
| 8 (A) ● (C) (D) (E) | 18 (A) (B) (C) ● (E) | 28 ● (B) (C) (D) (E) | 38 (A) (B) (C) (D) (E) | 48 (A) (B) (C) (D) (E) |
| 9 (A) (B) ● (D) (E) | 19 (A) (B) (C) ● (E) | 29 (A) (B) (C) (D) ● | 39 (A) (B) (C) (D) (E) | 49 (A) (B) (C) (D) (E) |
| 10 (A) (B) (C) ● (E) | 20 (A) (B) (C) (D) ● | 30 (A) ● (C) (D) (E) | 40 (A) (B) (C) (D) (E) | 50 (A) (B) (C) (D) (E) |